Capital Market Frictions, Business Cycle and Monetary Transmission

O. Pierrard

May 3, 2005

Abstract
Empirical evidence shows that some firms may be capital constrained because of capital market imperfections, and that these constraints may also affect employment creation. We therefore extend the business cycle models with frictions à la Pissarides on the labour market by also introducing and linking symmetric frictions on the capital market. We show that the capital market frictions (and their interactions with the labour market frictions) improve the statistical properties of the model and generate a financial accelerator.

Keywords: capital market frictions, business cycle, monetary transmission, financial accelerator

JEL classification: E13, E24, E51

*We would like to thank W. Bolt, P. van Els, C. Folkertsma, G. Nicoletti and E. Wasmer for valuable comments on this paper, as well as seminar participants at DNB and UQuaM. The views in the paper represent those of the authors and not necessarily those of the Bank.

†Research Division, De Nederlandsche Bank. Contact: Olivier Pierrard, De Nederlandsche Bank, Research Division, P.O. Box 98, 1000 AB Amsterdam, The Netherlands; phone: +31 20 524 3514; email: o.c.r.pierrard@dnb.nl.
1 Introduction

It is a well recognised fact that the labour market is characterised by imperfections: for instance, searching for workers is costly and may take time, just as searching for jobs is. Merz (1995), Andolfatto (1996) or den Haan, Ramey, and Watson (2000) nest this type of search frictions à la Pissarides (2000) into a dynamic general equilibrium (hereafter DGE) model and study their cyclical implications.

There also exist capital market imperfections. Several empirical papers conclude that it is difficult for some firms to find capital. Using US data sets, Himmelberg and Petersen (1994) or Gilchrist and Himmelberg (1999) show that the firm size especially matters to explain the capital constraints. And as noted by Lensink, Bo, and Sterken (2001):

"With respect to the size criterion it is generally assumed that larger firms are faced with less capital market imperfections than smaller firms since banks have fewer problems in screening and monitoring them."

Finally, Fahrer and Simon (1994) and Blanchflower (2004) emphasize that employment creation may also be constrained or delayed by lack of immediately available capital.

However, these capital market imperfections are rarely introduced into DGE models although it could help to answer some important questions: How the capital market imperfections (and their interactions with labour market frictions) affect the economy’s cyclical properties? And how these imperfections affect the monetary transmission mechanism?

This paper aims at studying these questions within a dynamic general equilibrium framework. Our starting point is the Merz (1995) and Andolfatto (1996) papers, that incorporate frictions on the labour market into a real business cycle model. As in Pissarides (2000), they represent the frictions by a matching function between labour demand (vacancies) and labour supply (unemployed). In their models, the capital market is however perfectly competitive. We therefore extend their framework by also adding imperfections on the capital market. To do so, and match empirical features, we make the distinction between large firms and small firms. Large firms (representative firms) are not risky (null destruction probability) and can directly obtain capital from the households through a perfect capital market (as in the above-mentioned models). Small firms (one-job-one-firm) are risky (positive destruction probability) and can only obtain capital from banks through an imperfect capital market (called hereafter financial market to avoid confusion with the perfect capital market).

Rather than to rely on moral hazard problems as in Bernanke, Gertler, and Gilchrist (1999) to explain the financial imperfections, we instead introduce these imperfections (and their interactions with the labour frictions) as in Was-
mer and Weil (2004): (i) financial imperfections are similar to the labour fric-
tions (we therefore have a localisation problem rather a moral hazard problem),
(ii) to produce, a small firm needs one unit of capital and one unit of labour
(one worker) and (iii) a small firm must first obtain capital before entering the
labour market to search for a worker. The advantage of this approach is to avoid
the complicated micro-modelisation associated to moral hazard problems and
to directly link financial imperfections and labour creation. Moreover, as men-
tioned in Wasmer and Weil (2004), there are several pieces of empirical evidence
that justify localisation problems to explain financial market imperfections.

We show that adding financial frictions to existing models with only labour
frictions improves the quantitative properties of the model. More specifically,
we obtain as in real data that consumption leads the cycle, investment is con-
temporaneous and employment lags. The intuition for this last result is that it
takes some time to hire people since the firms have first to find capital through a
frictional financial market. We also show that through the bank lending channel
(or narrow credit channel), a monetary shock (we assume a monetary injection
to the banks) stimulates employment, consumption, investment and output in
the short run, but is neutral in the long run. Finally, we quantitatively point
out that the interactions between the financial and the labour market frictions
generate a financial accelerator, that is they amplify and propagate the macroe-
conomic shocks (see also Wasmer and Weil (2004) for a similar observation
but in a static framework). We therefore recover the Bernanke, Gertler, and
Gilchrist (1999) result, but now triggered by localisation problems (for both the
financial and labour markets) rather than moral hazard problems.

The rest of the paper is organised as follows. Our model is presented in section 2
and calibrated in section 3. Section 4 proposes the results of different simulation
exercises. Section 5 concludes.

2 Model

There are four types of agents in our economy: small firms, large representa-
tive firms, representative banks and representative households. We assume that
a small firm needs one unit of capital and one worker to be able to produce
one intermediate goods. The capital for small firms is only provided by banks
through an imperfect capital market. Once the firm has found the capital, it
searches for a worker through an imperfect labour market. Finally, when the
firm produces, it pays a rent to the bank (resp. household) for the use of cap-
ital (resp. labour). We use the Pissarides (2000) representation of the labour
market, that consists of a two sided search market between firms and represen-
tative households-workers: it is difficult to locate labour supply/demand and it
is costly and time consuming for a firm (resp. a worker) to search for a worker
(resp. a firm). Following den Haan, Ramey, and Watson (2003) and Wasmer
and Weil (2004), the financial market is modeled in a fully symmetric way, i.e. it
consists of a two sided search market between firms and representative banks: it
is difficult to locate capital supply/demand and it is costly and time consuming for a firm (resp. a bank) to search for a bank (resp. a firm). In both cases, the frictions may arise from information problems and geographical distance. The frictions on both the financial market and the labour market are represented by standard matching functions. The small firm capital rent is determined as a share of the surplus generated by the production, and we assume an exogenous wage.

We also have large firms using the intermediate goods as well as capital to produce final goods. The capital for large firms is directly provided by the households. The final and intermediate goods markets and the large firms capital market are perfectly competitive. The price of the final goods is normalised to 1, the price of the intermediate goods adjusts to clear the intermediate goods market and the interest rate adjusts to clear the capital market.

Finally, we have a central bank directly injecting money into the representative banks.

### 2.1 Financial and labour frictions and flows

If $L_t$ is the new capital supply (by the banks) and $E_t$ the new capital demand (by the small firms), we define the number of new capital matches by $H(L_t, E_t)$, where $H$ is a matching function satisfying the usual Inada conditions. The total population is normalised to 1 and can be employed ($N_t$) or unemployed ($1 - N_t$).

If $V_t$ is the new labour demand (by the firms) and $1 - N_t$ the new excess labour supply (by the households), we define the number of new labour matches by $M(V_t, 1 - N_t)$, where $M$ is a matching function satisfying the usual Inada conditions.

We assume that capital and labour matches are destroyed with the exogenous probability $s$. The dynamic flows on the financial and labour markets are:

\[
V_{t+1} = (1 - s)V_t - M(V_t, 1 - N_t) + H(L_t, E_t),
\]

\[
N_{t+1} = (1 - s)N_t + M(V_t, 1 - N_t).
\]

The probability $p_t^B$ for a bank to find a firm and the probability $p_t^F$ for a firm to find a bank are:

\[
p_t^B = \frac{H(L_t, E_t)}{L_t} \quad \text{and} \quad p_t^F = \frac{H(L_t, E_t)}{E_t}.
\]

The probability $q_t^F$ for a firm to find a worker and the probability $q_t^H$ for a

---

1Shimmer (2005) shows on US data that the separation probability is nearly acyclical, particularly during the last decades. Hall (2005) also emphasizes that for the past 50 years in the US, the separation rate is nearly constant while the job-finding rate shows high volatility at business cycle.
worker to find a firm are:

\[ q_t^F = \frac{M(V_t,1-N_t)}{V_t} \quad \text{and} \quad q_t^H = \frac{M(V_t,1-N_t)}{1-N_t}, \]  

(4)

2.2 Small firms

The small firms can be in three different states:

- **state 1**: firms searching for a bank able to provide one unit of capital
- **state 2**: firms with the unit of capital searching for one worker
- **state 3**: firms with one unit of capital and one worker, producing one intermediate goods and paying back a capital rent to the bank and a labour rent (wage) to the household

The three states’ asset values are:

\[ W_{t}^{F,1} = -c + \bar{\beta}_t E_t \left[ (1-p_t^F)W_{t+1}^{F,1} + p_t^F W_{t+1}^{F,2} \right], \]  

(5)

\[ W_{t}^{F,2} = -\gamma + \bar{\beta}_t E_t \left[ sW_{t+1}^{F,1} + (1-q_t^F - s)W_{t+1}^{F,2} + q_t^F W_{t+1}^{F,3} \right], \]  

(6)

\[ W_{t}^{F,3} = -\rho_t - w_t + d_t + \bar{\beta}_t E_t \left[ sW_{t+1}^{F,1} + (1-s)W_{t+1}^{F,3} \right], \]  

(7)

where \( c \) is the firm cost of searching for a bank (capital demand cost), \( \gamma \) is the firm cost of searching for a worker (labour demand cost), \( \rho_t \) is the capital price paid to the bank, \( w_t \) is the wage paid to the worker, \( d_t \) is the price of the intermediate goods and \( \bar{\beta}_t \) is the rate at which future profits are discounted (see section 2.5). The free entry condition states:

\[ W_{t}^{F,1} = 0. \]  

(8)

2.3 Large representative firm

The representative firm’s asset value satisfies the Bellmann equation:

\[ W_{t}^{FF} = \max_{N_t,K_{t+1}} \left\{ \varepsilon_t F(K_t,N_t) - d_t N_t - (r_t + \delta) K_t + \bar{\beta}_t E_t \left[ W_{t+1}^{FF} \right] \right\}, \]  

(9)

where \( \varepsilon_t \) is an aggregate productivity shock, \( F \) is a production function satisfying the usual Inada conditions, \( N_t \) is the amount of intermediate goods (or equivalently the amount of small firms producing, or equivalently the amount of employed workers), \( K_t \) is the capital stock, \( r_t \) is the capital interest rate paid to the household, \( \delta \) is the depreciation rate of capital. Maximising (9) with respect to \( N_t \) and \( K_{t+1} \) gives respectively:

\[ \varepsilon_t \frac{\partial F(K_t,N_t)}{\partial N_t} = d_t, \]  

(10)

\[ \varepsilon_t \frac{\partial F(K_t,N_t)}{\partial K_t} = r_t + \delta, \]  

(11)
2.4 Representative bank

The representative bank’s asset value satisfies the Bellmann equation:

\[ W_t^B = \max_{L_{t+1}} \left\{ \rho_t N_t - k L_{t+1} + \tilde{\beta}_t E_t \left[ W_{t+1}^B \right] \right\}, \]  \hspace{1cm} (12)

under the flow constraints:

\[ V_{t+1} = (1 - s - q^F_t) V_t + p_t^B L_t, \]  \hspace{1cm} (13)

\[ N_{t+1} = (1 - s) N_t + q^F_t V_t, \]  \hspace{1cm} (14)

where \( k \) is the bank cost of searching for a firm (capital supply cost). It is worth noting that the banks choose today the capital supply of tomorrow (predetermined capital supply). This allows to generate real effects after a monetary shock (see section 2.7). Maximising (12) with respect to \( L_{t+1} \) and under the constraints (13) and (14) gives:

\[ -k + \tilde{\beta}_t E_t \left[ \frac{\partial W_{t+1}^B}{\partial L_{t+1}} \right] = 0. \]  \hspace{1cm} (15)

By the envelope theorem, we also have:

\[ \frac{\partial W_t^B}{\partial L_t} = \tilde{\beta}_t E_t \left[ \frac{\partial W_{t+1}^B}{\partial V_{t+1}} \right], \]  \hspace{1cm} (16)

\[ \frac{\partial W_t^B}{\partial V_t} = \tilde{\beta}_t (1 - s - q^F_t) E_t \left[ \frac{\partial W_{t+1}^B}{\partial V_{t+1}} \right] + \tilde{\beta}_t q^F_t E_t \left[ \frac{\partial W_{t+1}^B}{\partial N_{t+1}} \right], \]  \hspace{1cm} (17)

\[ \frac{\partial W_t^B}{\partial N_t} = \rho_t + \tilde{\beta}_t (1 - s) E_t \left[ \frac{\partial W_{t+1}^B}{\partial N_{t+1}} \right]. \]  \hspace{1cm} (18)

2.5 Representative household

The representative household’s welfare satisfies the Bellmann equation:

\[ W_t^H = \max_{K_{t+1}} \left\{ \mathcal{U}(C_t) + \beta E_t \left[ W_{t+1}^H \right] \right\}, \]  \hspace{1cm} (19)

under the budget constraint:

\[ C_t + I_t = N_t w_t + (r_t + \delta) K_t + \Pi_t^F + \Pi_t^B, \]  \hspace{1cm} (20)

where \( \mathcal{U} \) is an increasing and concave utility function, \( C_t \) is consumption, \( I_t \) is investment and \( \beta \) is the household discount factor. Investment is defined as:

\[ I_t = K_{t+1} - (1 - \delta) K_t. \]  \hspace{1cm} (21)

We assume that the households hold the small firms and the banks and therefore receive their whole profits, respectively \( \Pi_t^F \) from the firms and \( \Pi_t^B \) from the
banks. Maximising (19) with respect to $K_{t+1}$ and under the constraint (20) gives:

$$\frac{\partial U(C_t)}{\partial C_t} = \beta E_t \left[ (1 + r_{t+1}) \frac{\partial U(C_{t+1})}{\partial C_{t+1}} \right].$$

(22)

Since both the small firms and the banks are held by the households, the rate at which future profits are discounted is the same for the small firms and the banks:

$$\tilde{\beta}_t = \beta E_t \left[ \frac{\partial U(C_{t+1})}{\partial C_{t+1}} \frac{\partial C_t}{\partial U(C_t)} \right].$$

(23)

2.6 Labour and capital prices

By simplicity, we assume that wages are exogenous:\2:\

$$w_t = \bar{w}.$$ \hspace{1cm} (24)

We assume that the capital price $\rho_t$ is Nash bargained between the bank and the small firm once the firm starts to produce\3:\, that is:

$$\max_{\rho_t} \left( W_{t,F}^F - W_{t,F}^F,1 \right)^{\frac{\zeta}{1-\zeta}} \left( \frac{\partial W_{t,F}^B}{\partial N_t} \right)^{1-\zeta},$$

(25)

where $0 < \zeta < 1$ is the firm’s bargaining power. Maximising (25) with respect to $\rho_t$ gives\4:\

$$(1-\zeta) \left( \frac{\partial W_{t,F}^B}{\partial N_t} + W_{t,F}^F,3 - W_{t,F}^F,1 \right) = \frac{\partial W_{t,F}^B}{\partial N_t}$$

(26)

2.7 Central bank and monetary policy

We assume that the central bank may inject money into the financial market (through the banks). Then the new capital supply $L_t$ is defined as:

$$L_t = L_p^B + \frac{M_t}{P_t},$$

(27)

where $L_p^B$ is the - predetermined - bank own capital supply, $M_t$ is the central bank monetary injection and $P_t$ is the price. As in Cooley and Hansen (1989),

\footnote{This assumption is consistent with the fact that in real data, wages are mostly acyclical and have a low volatility, see for instance King and Rebelo (1999) for a US empirical evidence or Shimmer (2004) for simulation results. In appendix 1, we nevertheless assume a Nash bargained wage and compare the two economies (exogenous vs. Nash bargained wage).}

\footnote{We transpose to the financial market a usual bargaining rule for the labour market.}

\footnote{Under certain circumstances (too high wages combined with strongly negative shocks), firms and banks could want to destroy the job. By assuming a sufficiently high firing tax, we show that this case can be avoided. See appendix 2 for a formal demonstration.}
we assume that consumption is a cash good while investment is a credit good. The price is therefore determined by the cash constraint\(^5\):

\[ C_t = \frac{\bar{M} + M_t}{P_t}, \]  

(28)

where \( \bar{M} \) is the initial money stock. Finally, the maximisation program (12) becomes:

\[ W_t^R = \max_{L_{t+1}} \left\{ p_t N_t - k(L_t^P + \frac{M_t}{P_t}) + \tilde{\beta}E_t [W_{t+1}^R] \right\}. \]  

(29)

3 Calibration

We use the following specific functions:

\[ H(L_t, E_t) = m^H L_t^{\lambda_H} E_t^{1-\lambda_H}, \]  

(30)

\[ M(V_t, 1 - N_t) = m^M V_t^{1-\lambda_M} (1 - N_t)^{\lambda_M}, \]  

(31)

\[ F(K_t, N_t) = K_t^{\mu} N_t^{1-\mu}, \]  

(32)

\[ U(C_t) = \ln(C_t). \]  

(33)

The two matching functions as well as the production function are Cobb-Douglas with constant return to scale, and the utility function is logarithmic.

We calibrate our model on quarterly data and to reproduce some stylised facts for a representative US economy. As in most business cycle models, the psychological discount factor \( \beta \) is set to 0.99, implying a steady state real interest rate of 4% per annum. Parameter \( \mu \) is the elasticity of production with respect to capital and is set to its standard value 0.33. The depreciation rate \( \delta \) is set to 0.025 and implies a steady state capital-production ratio of 9. As in most of the labour matching literature, the elasticity \( \lambda_M \) of job matches with respect to the job seekers is fixed at 0.5. Following Wasmer and Weil (2004), we also set the elasticity \( \lambda_H \) of capital matching with respect to capital supply to 0.5, as well as the sharing parameter \( \zeta \). Shimer (2005) estimates an average (over the last 55 years) US job destruction rate slightly below 4% and we therefore impose \( s = 0.04 \).

The matching efficiencies, the capital search costs, the labour search cost and the wage \( (m^H, m^M, c, k, \gamma, \bar{w}) \) are chosen to reproduce the observed values for the following 6 variables. We assume an unemployment rate of 7% \( (1 - N = 0.07) \), which is approximately the average US unemployment rate over the last 30 years (see for instance OECD data). By simplicity, we assume all the endogenous probabilities \( p^F = q^F = p^B = q^H = 0.5 \). This means that on average, it takes

\( ^5 \text{We do not have to introduce a cash in advance constraint since the } L_t^P \text{ predetermination is already sufficient to generate monetary policy real effects.} \)
2 quarters to obtain capital from a bank, to recruit a worker, to supply capital to a firm and to find a job\(^6\).

Finally, the steady state value \(\varepsilon\) of the productivity shock is normalised to 1. We assume that there is no monetary injections at the steady state \((M = 0)\) and the money stock \(\bar{M}\) is chosen to normalise the steady state price at 1. The numerical values of the calibrated parameters are reported in table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta)</td>
<td>0.025</td>
<td>(\beta)</td>
<td>0.99</td>
</tr>
<tr>
<td>Production function</td>
<td>(\varepsilon)</td>
<td>(\mu)</td>
<td>0.33</td>
</tr>
<tr>
<td>Matching functions</td>
<td>(m_H)</td>
<td>(\lambda^H)</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(m_M)</td>
<td>(\lambda^M)</td>
<td>0.50</td>
</tr>
<tr>
<td>Capital and labour prices</td>
<td>(\zeta)</td>
<td>(\bar{w})</td>
<td>1.98</td>
</tr>
<tr>
<td>Search costs and job destruction rate</td>
<td>(k)</td>
<td>(c)</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(\gamma)</td>
<td>(s)</td>
<td>0.04</td>
</tr>
<tr>
<td>Money</td>
<td>(\bar{M})</td>
<td>2.11</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Numerical parameter values

4 Simulations

In this section, we discuss through different simulation exercises if the introduction of the imperfect capital market quantitatively improves the statistical properties of business cycle models.

4.1 Aggregate productivity shock

We first assume that economic fluctuations are driven by a stochastic aggregate productivity shock:

\[
\varepsilon_t = \varepsilon^{1-n} \varepsilon_{t-1} \varepsilon^{u_t},
\]

\(^6\)We could try to estimate more precise probabilities (although only few empirical data are available to estimate the financial market probabilities) but this would not affect our simulation results.
where $\eta$ is the autoregressive parameter and $u^1_t \sim N(0, \sigma^2_{u^1})$. We set $\eta = 0.9$ and $\sigma_{u^1} = 0.005$.

**Cyclical properties**

We simulate three different models: (i) a basic RBC model à la King, Plosser, and Rebelo (1988) where all markets are perfectly competitive, (ii) a RBC model à la Merz (1995) with frictions on the labour market (model that nests labour market frictions à la Pissarides (2000) into the basic RBC model) and (iii) our model with frictions on both the financial and the labour markets (model that nests the financial and labour frictions à la Wasmer and Weil (2004) into the basic RBC model). We calibrate similarly all the models and we compare the simulation results to the business cycle characteristics of US data (the source and methodology are reported in appendix 3). The simulation results as well as the US stylised facts are summarized in table 2.

By comparing the two first parts of table 2, we see that three shortcomings (if compared to real data) of the basic RBC model are: (i) there is no room for unemployment (there is only an endogenous choice between work or leisure), (ii) there is not enough persistence in output, unless we introduce an - unrealistic - very highly autocorrelated productivity shock, and (iii) all the variables are perfectly contemporaneous, although consumption leads and employment lags output in real data.

Introducing frictions on the labour market helps to overcome the first two problems: frictional unemployment is introduced and the labour market dynamics add persistence in data. We also see that consumption and employment/unemployment now leads output, which is empirically relevant for consumption but counterfactual for unemployment. The intuition is that creating employment is costly and that offsets, during the first period following the shock, the direct positive impact on output of the productivity shock$^7$.

Our model retains the main properties of the previous one but also allows to correct this employment problem. Creating employment now first necessitates to obtain capital from a bank. The employment creation process is therefore much slower and that explains why employment/unemployment now lags output. Moreover, we remarkably match the unemployment cross-correlation empirical figures with both lag and lead GDP. It is worth noting that in the two models with frictions, we impose an exogenous wage. This is justified by the fact that in real data, the wage volatility is quite low, as well as the wage-output correlation (see for instance Andolfatto (1996) or King and Rebelo (1999) for empirical evidences). But since the wage adjustment is by definition reduced to

$^7$Formally: $\text{GDP} \equiv C_t + I_t = \varepsilon_t F(K_t, N_t) - \gamma V_t$. An increase in $\varepsilon_t$ generates a current surge in $V_t$ and the effect on GDP is ambiguous.
Table 2: Business cycle statistics
zero, we have an overreaction to shocks of the unemployment level\(^8\).

Financial accelerator

In some cases, endogenous developments in capital markets amplify macroeconomic shocks and therefore act as a financial accelerator. In Bernanke, Gertler, and Gilchrist (1999), moral hazard problems create a wedge between external and internal finance and generate a risk premium. A positive macro shock increases the value of the collateral and then reduces the moral hazard problem and the risk premium, that in turn amplify the effects of the macro shock. In our model, endogenous developments in the capital market also act as a financial accelerator, but through a completely different mechanism. A positive macro shock stimulates labour demand and thus capital demand. This generates a positive externality for the banks (higher probability to get matched) that increases their capital supply. This in turn generates a positive externality for the firms that further stimulates labour demand. In this case, the financial accelerator is generated by macroeconomic interactions between imperfect capital and labour markets.

This financial accelerator is numerically illustrated in figure 2. We represent the impulse response functions to an aggregate productivity shock in our benchmark economy, and in an economy where there is less frictions on the financial market (more precisely, we increase the parameter \(m^H\) from 0.5 to 0.6). We see that increasing the imperfections on the financial market (that is decreasing the above-mentioned parameter) amplifies and propagates the macroeconomic shock. Frictions on the financial markets (and their interactions with frictions on the labour market) therefore act as a financial accelerator (see also Wasmer and Weil (2004) for a similar evidence in a static model).

4.2 Monetary shock

We assume a permanent monetary supply shock \(M_t = M^s > 0\). This monetary injection by the central bank is directly supplied to the commercial banks. The effects of this permanent shock are reproduced in figure 2. Since the supply of own capital \(L_p^t\) by the banks is determined prior to the monetary shock, the shock increases the total capital supply during the first period, which stimulates labour demand, capital demand and, in fine, employment, investment and output (GDP). Our monetary transmission through this bank lending channel (also called narrow credit channel) also stimulates the real interest rate. After the first period, the commercial banks react to the monetary shock by adjusting downwards their own capital supply and the economy goes back progressively to its initial steady state. Our monetary shock therefore leads to short run positive effects but is neutral in the long run, the only long run effect being a permanent

---

\(^8\)An alternative modelisation is to introduce a Nash bargaining framework for the wage determination. This would however deteriorate our results, by drastically reducing the unemployment volatility and correlations. The model with a Nash bargained wage and its statistical properties are displayed in appendix 1.
Figure 1: Impulse responses to a positive aggregate productivity shock
increase in the price. Most of these results are coherent with VAR empirical
evidences (see for instance Christiano, Eichenbaum, and Evans (1999)).

In most of the dynamic general equilibrium models (see, among many oth-
ers, Yun (1996), Goodfriend and King (1997), Christiano, Eichenbaum, and
Evans (1997), Smets and Wouters (2003) or Christiano, Eichenbaum, and Evans
(2005)), the monetary transmission mechanism is realised through the interest
rate channel. Because of nominal rigidities (and possibly limited participation
constraints), a monetary expansion decreases the real interest rate and stimu-
lates investment and output. In our model, the transmission process is realised
through a pure bank lending channel, which itself results from frictions on the
financial market. This lack of nominal rigidities also explains why we do not
have a decrease in the interest rate in our model.

To show the importance of the frictions on the financial market, we conduct
the same simulations but assuming lower frictions (more precisely, we increase
the parameter $m^H$ from 0.5 to 0.6). We see that increasing the imperfections
on the financial market (that is decreasing the above-mentioned parameter)
amplifies and propagates the macroeconomic shock. As displayed in figure 2
and already mentioned for the productivity shock, we see that frictions on the
financial markets (and their interactions with frictions on the labour market)
act as a financial accelerator. In other words, the higher are the frictions on the
financial market, the more important is the monetary transmission through the
bank lending channel.

### 4.3 Variance decomposition

In our stylized model, the real effects of the monetary shocks are fully due to
the predetermination of the bank capital supply. Although it is sufficient to
generate realistic impulse response functions and a financial accelerator, a mon-
etary shock has much less impact on the real economy than a real productivity
shock of similar amplitude. By looking at a variance decomposition, we see
that the monetary shock accounts for less than 1% to the variation of the GDP
components and employment (see table 3). This is understandable since there
is no room for nominal rigidities in our benchmark model.

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>C</th>
<th>I</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.2%</td>
<td>0.1%</td>
</tr>
<tr>
<td>exo nominal wage</td>
<td>3.8%</td>
<td>3.2%</td>
<td>4.7%</td>
<td>41.1%</td>
</tr>
</tbody>
</table>

GDP = output = C + I, C: consumption, I: investment, N: employment

Table 3: Variance decomposition: effect of the money supply shock

To get more insights about the role of nominal rigidities on the monetary trans-
Figure 2: Responses to a positive permanent monetary shock
mission mechanism, we now assume an exogenous nominal wage. Formally, we replace equation (24) by:

\[ w_t = \bar{w} \frac{P_t}{P_t} \]  

(35)

In this case, by increasing the price, a monetary injection lowers the real wage and this further stimulates employment and the GDP components (see last line of table 3).

5 Conclusion

In this paper, we extend the Merz (1995) and Andolfatto (1996) RBC models with frictions on the labour market, by also introducing imperfections on the capital market. To do so, rather than to rely on asymmetric information and moral hazard problems as in Bernanke, Gertler, and Gilchrist (1999), we rely on localisation problems as in Wasmer and Weil (2004) and model the frictions on the capital market in a fully symmetric way to the frictions on the labour market. This modelisation is empirically justified and has the advantage of a simple and elegant formulation.

Our main results are that (i) the introduction of the imperfect capital market improves the cyclical properties of the model and especially the employment/unemployment behaviour, (ii) through a bank lending monetary transmission mechanism, our simple model reproduces the positive short run effects of a monetary shock and its long run neutrality, and (iii) the frictions on the financial market act as a financial accelerator, that is they amplify and propagate the macroeconomic shocks.

To keep this simple and - we hope - elegant formulation, our model is very sylised and could then be developed along several dimensions:

- Neither the exogenous wage (no flexibility at all) nor the Nash bargained wage (too much flexibility) lead to a fully convincing modelisation. Making a difference between outsiders and insiders, or introducing a Calvo type wage in the search framework could improve our modelisation. Moreover, introducing other nominal (Calvo price) or real (habit formation, capital adjustment cost,...) rigidities as in recent New-Keynesian DSGE models would add more realism to the model.

- We have a "one-sided" banking sector: banks have capital on their own and decide to keep it (no cost but no income) or to supply it (costly but expected income). Extending this framework by allowing banks to receive deposit from households would be a natural extension. A monetary shock would in this case also directly affect the households, rather that only in an indirect way.

- A lot of empirical papers try to assess the relative importance of the different monetary transmission channels (see for instance van Els, Locarno,
Mojon, and Morgan (2003) for a recent survey). Adding a "classical" interest rate channel on top of the bank lending channel could shed a light on this question.

All these extensions would however complicate this stylised model and we leave them for future research.
References


Appendix 1: Nash bargained wage

If we assume a Nash bargained wage, equation (24) becomes:

$$\max_{w_t} \left( W_{t}^{F,3} - W_{t}^{F,1} \right) \xi \left( \frac{\partial W_t^H}{\partial N_t} \frac{\partial C_t}{\partial H(C_t)} \right)^{1-\xi}$$

(36)

where

$$\frac{\partial W_t^H}{\partial N_t} = \frac{\partial H(C_t)}{\partial C_t} w_t + \beta (1 - s - q^H_t) E_t \left[ \frac{\partial W_{t+1}^H}{\partial N_{t+1}} \right]$$

(37)

and $0 < \xi < 1$ is the firm’s bargaining power. The maximisation program gives:

$$(1 - \xi) \left( \frac{\partial W_t^H}{\partial N_t} + \frac{\partial H(C_t)}{\partial C_t} (W_t^{F,3} - W_t^{F,1}) \right) = \frac{\partial W_t^H}{\partial N_t}$$

(38)

We simulate this model, where $\xi = 0.1$ in order to obtain a steady state wage equal to $\bar{w}^9$. Table 4 compares these simulations to our benchmark model with exogenous wage and to the empirical facts. We see that adding the bargained wage deteriorates our results, by drastically reducing the unemployment volatility and correlations.

---

9Comparing to usual estimations (between 0.4 and 0.6) this gives a quite low firm bargaining power. Increasing this bargaining power would however not much affect our simulation results.

<table>
<thead>
<tr>
<th>Variables</th>
<th>st. dv. rel/GDP</th>
<th>cross corr. with GDP(t+k)</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.00</td>
<td>0.29</td>
<td>0.88</td>
</tr>
<tr>
<td>consumption</td>
<td>0.80</td>
<td>0.10</td>
<td>0.69</td>
</tr>
<tr>
<td>investment</td>
<td>2.84</td>
<td>0.36</td>
<td>0.88</td>
</tr>
<tr>
<td>unemployment</td>
<td>7.44</td>
<td>-0.45</td>
<td>-0.90</td>
</tr>
</tbody>
</table>

### Model with labour and financial frictions, exogenous wage

<table>
<thead>
<tr>
<th>Variables</th>
<th>st. dv. rel/GDP</th>
<th>cross corr. with GDP(t+k)</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.00</td>
<td>0.36</td>
<td>0.90</td>
</tr>
<tr>
<td>consumption</td>
<td>0.41</td>
<td>0.32</td>
<td>0.70</td>
</tr>
<tr>
<td>investment</td>
<td>3.14</td>
<td>0.34</td>
<td>0.90</td>
</tr>
<tr>
<td>unemployment</td>
<td>12.4</td>
<td>-0.64</td>
<td>-0.88</td>
</tr>
</tbody>
</table>

### Model with labour and financial frictions, Nash bargained wage

<table>
<thead>
<tr>
<th>Variables</th>
<th>st. dv. rel/GDP</th>
<th>cross corr. with GDP(t+k)</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.00</td>
<td>0.10</td>
<td>0.71</td>
</tr>
<tr>
<td>consumption</td>
<td>0.29</td>
<td>0.44</td>
<td>0.77</td>
</tr>
<tr>
<td>investment</td>
<td>3.45</td>
<td>0.01</td>
<td>0.66</td>
</tr>
<tr>
<td>unemployment</td>
<td>0.43</td>
<td>-0.88</td>
<td>-0.39</td>
</tr>
</tbody>
</table>

HP filtered quarterly data. US stylised facts: source and methodology in appendix 3. Simulated data: GDP, or output, is defined as the sum of consumption and investment.

Table 4: Business cycle statistics
Appendix 2: Job destruction and firing tax

A match (job) involves 3 agents: the worker, the firm and the bank. In our setting, an exogenous wage is paid to the worker and the remaining surplus is shared between the bank and the firm, following equation (26):

\[ W_{F,3}^t = \zeta \left( d_t - \bar{w} + \tilde{\beta}_t (1 - s) E_t \left[ W_{t+1}^{F,3} + W_{N,t+1}^B \right] \right) , \tag{39} \]

\[ W_{N,t}^B = (1 - \zeta) \left( d_t - \bar{w} + \tilde{\beta}_t (1 - s) E_t \left[ W_{t+1}^{F,3} + W_{N,t+1}^B \right] \right) . \tag{40} \]

As long as \( W_{F,3}^t \geq 0 \) and \( W_{N,t}^B \geq 0 \), i.e. as long as:

\[ d_t - \bar{w} + \tilde{\beta}_t (1 - s) E_t \left[ W_{t+1}^{F,3} + W_{N,t+1}^B \right] \geq 0 , \tag{41} \]

there is no incentive for the firm and the bank to destroy the match\(^{10}\). We notice that for high \( \bar{w} \) and/or low \( d_t \) (bad aggregate productivity shock), this condition may be violated.

We now assume that the firm and the bank have the choice to continue or to destroy the match and pay a firing tax \( f \). Asset values (7) and (18) can now be re-written, respectively:

\[ W_{F,3}^t = -\rho_t - w_t + d_t + \tilde{\beta}_t (1 - s) E_t \left[ \max \left\{ W_{t+1}^{F,3} + W_{N,t+1}^B, -f \right\} \right] , \tag{42} \]

\[ W_{N,t}^B = \rho_t + \tilde{\beta}_t (1 - s) E_t \left[ \max \left\{ W_{N,t+1}^B, -f \right\} \right] . \tag{43} \]

The usual sharing rules (39) and (40) now become, respectively:

\[ W_{F,3}^t + f = \zeta \left( d_t - \bar{w} + 2f + \tilde{\beta}_t (1 - s) E_t \left[ \max \left\{ W_{t+1}^{F,3} + W_{N,t+1}^B, -2f \right\} \right] \right) , \]

\[ W_{N,t}^B + f = (1 - \zeta) \left( d_t - \bar{w} + 2f + \tilde{\beta}_t (1 - s) E_t \left[ \max \left\{ W_{t+1}^{F,3} + W_{N,t+1}^B, -2f \right\} \right] \right) . \]

As long as \( W_{F,3}^t + f \geq 0 \) and \( W_{N,t}^B + f \geq 0 \), i.e. as long as:

\[ d_t - \bar{w} + 2f + \tilde{\beta}_t (1 - s) E_t \left[ \max \left\{ W_{t+1}^{F,3} + W_{N,t+1}^B, -2f \right\} \right] \geq 0 , \tag{44} \]

there is no incentive for the firm and the bank to destroy the match\(^{11}\). It is easy to show that by taking \( f \) large enough, inequality (44) is never violated.

In this case, equation (42) is equivalent to equation (7), equation (43) is equivalent to equation (18), and the sharing rules are:

\[ W_{F,3}^t = \zeta \left( d_t - \bar{w} + \tilde{\beta}_t (1 - s) E_t \left[ W_{t+1}^{F,3} + W_{N,t+1}^B \right] \right) + f(2\zeta - 1) , \tag{45} \]

\(^{10}\)We see from equations (39) and (40) that it is a joint decision.

\(^{11}\)Note that we also have a joint decision.
\[ W_{N_t}^B = (1 - \zeta) \left( d_t - \bar{w} + \tilde{\beta}_t (1 - s) E_t \left[ W_{t+1}^{E,3} + W_{N_{t+1}}^B \right] \right) - f(2\zeta - 1). \] (46)

With our calibration, \( \zeta = 1/2 \), and equation (45) is equivalent to equation (39), equation (46) is equivalent to equation (40). We therefore have a strict equivalence between the model with the high firing tax and the model exposed in the main text.

**Appendix 3: Description of the data source and methodology**

US data used in the paper include GDP, Consumption, Investment and Unemployment rate. The first three series come from the Quarterly National Accounts database of the OECD, from 1970q1 to 2004q4. All data are transformed into their logarithm and HP filtered with a 1600 weight.

The last series comes from the OECD Main Economic Indicator database, from 1970m1 to 2004m12. This is a monthly series and we take the three months average value for the quarter of interest. Unemployment rate data are HP filtered with a 1600 weight.