Early Literacy Achievements, Population Density and the Transition to Modern Growth

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Human capital theories: quality of labor $\rightarrow$ growth

Link between literacy in the pre-industrial era and the process leading to the Industrial Revolution?

For Cipolla (1969) early improvements in literacy paved the way for the Industrial Revolution.

In England, improvements in literacy started as early as in the sixteenth century.
Literacy achievements (% population) [Estimation: Cressy (1980).]
Explanatory factors?

First: productivity gains in the modern sector increased the return to investment in education

but timing is wrong: Little productivity gains before 1700
Total factor productivity [Estimation: Clark 2001]
Second explanation: longevity improvements.

Adult mortality started to drop before the Industrial Revolution (Boucekkine et al. 2003).

Lower mortality increases the return to education and may have induced higher investment in human capital.

Problem with England: longevity was actually stagnant over the period 1500 to 1700 or even declining after 1600
Mortality: number of survivors at age 40 from 1000 individuals at age 10 [Source: Wrigley et al. (1997)]
Third explanation: Higher density of population stimulated technical progress.

Becker et al. (1999): larger populations encourage greater specialization and increased investments in knowledge.

Galor and Weil (1998): “population-induced” technical progress which raised the return to human capital.
Population of England [Estimation: Wrigley et al. (1997)]
In Galor and Weil’s papers the effect of population on productivity is assumed instead of being derived from primary assumptions.

We want to derive the effect of population on productivity from some maximizing behavior.

More precisely, both the number and location of education facilities is chosen optimally.

Higher population density makes it optimal to multiply the number of schools, opening the possibility to reach higher educational levels.
Time and space

Time is continuous. At each point in time a new generation of size $\zeta_t$ is born.

Individuals born at the same date have different innate abilities, $\mu$, and location, $i$.

$\mu \in [0, \bar{\mu}]$
Space: circle of unit length.

Each new generation is uniformly spread over the circle.

Schools are optimally located if they are evenly spaced, let us say at locations $j/E$, where $j = 0, \ldots, (E - 1)$.

$x(i)$: distance between the individual located at $i$ and the closest school. If no school is created, distance is infinite:

$$x(i) \in \left[0, \frac{1}{2}\right] \cup \{+\infty\}.$$
Demographics

Survival function:

\[ m_t(a) = \frac{e^{\beta_t a} - \alpha_t}{1 - \alpha_t}, \]  \hspace{1cm} (1)

Maximum age:

\[ L_t = \frac{\log(\alpha_t)}{\beta_t}. \]  \hspace{1cm} (2)
Size of the generation born in $t$ at any time $z \in [t, t + L_t]$

$$\zeta_t \, m_t(z - t).$$

(3)

The size of total population at time $t$ is given by

$$P_t = \int_{t - L_t}^{t} \zeta_z \, m_z(t - z)dz,$$

(4)
Material good, produced through two different technologies.

In the “modern sector”, the technology employs human capital $H_t$ with constant returns:

$$Y_t = A_t \ H_t \quad \text{where} \quad A_t = e^{\gamma t}.$$  \hfill (5)

In the “traditional sector”, individuals have a productivity $w^h$ per unit of time, independent of their level of human capital.
Individuals \((\mu, i)\) born at time \(t\)

Maximization of lifetime resources \(W\):

\[
W = \int_{t+S_t(\mu, i)}^{t+L_t} \omega_t(\mu, i, z) \, m_t(z - t) \, e^{-\theta(z-t)} \, dz \\
- \int_{t+S_t(\mu, i)}^{t} \xi x(i) \, e^{\gamma z} \, m_t(z - t) \, e^{-\theta(z-t)} \, dz - k \, e^{\gamma t} \, \delta(S_t(\mu, i)),
\]

where \(k\) is a fixed cost to be paid only if the individual decides to go to school:

\[
\delta(S_t(\mu, i)) = 1 \text{ if } S_t(\mu, i) > 0, \text{ and } = 0 \text{ otherwise.}
\]
Spot wage:

$$\omega_t(\mu, i, z) = h_t(\mu, i)A_z,$$

Education technology:

$$h_t(\mu, i) = \mu S_t(\mu, i). \quad (7)$$

For education to be an optimal outcome:

$$W > \int_t^{t+L_t} w^h m_t(z - t) e^{-\theta(z-t)} dz. \quad (8)$$
School location

At each date a number of classrooms is created to serve the newborn generation.

A central authority determines the optimal number of classrooms so as to maximize the profit of the whole school system.

Profits:

\[(kR(E_t) - f) E_t\]  \(\text{(9)}\)

where \(R(E_t)\) is the number of children attending one school.
Equilibrium

Given exogenous demographic and technological trends $\alpha_t, \beta_t, \gamma_t$ and $\zeta_t$, an equilibrium consists of

– A path of optimal education decision $\{S_t(\mu, i)\}_{t \geq 0}$ maximizing life-time resources (6), subject to (8);
– A path of optimal number of schools $\{E_t\}_{t \geq 0}$ maximizing the profits of the school system (9).

Resolution:
– the individual problem
– the education authorities problem
Solution to the individual's choice (for $\theta = 0$)

Existence and uniqueness of the interior solution

The first-order necessary condition is:

$$\mu \int_{t+L}^{t+S} A_z m_t(z-t)dz = m_t(S_t(\mu,i)) A_{t+S}(\mu S_t(\mu,i) + \xi x(i)).$$

(10)

Under a steady technological progress:

$$\mu \int_{S(\mu,i)}^{L} e^{\gamma a} m(a)da = (\mu S(\mu,i) + \xi x(i)) e^{\gamma S(\mu,i)} m(S(\mu,i)).$$

(11)
Assumption 1  The parameter $\mu$ satisfies:

$$\mu > \mu(i).$$

Proposition 1  For $\gamma$ small enough, there exists a solution to (11) such that $0 < S(\mu, i) < L$ if and only if Assumption 1 holds. The solution is unique. This solution tends to zero as $\mu$ gets closer to $\mu_i$. 
\[ \mu(i) = \frac{\xi x(i)}{\int_0^L e^{\gamma a} m(a) \, da} \]

**Corollary 1** The threshold \( \mu_i \) is an increasing function of \( \xi, x(i) \) and \( \beta \). It is decreasing in \( \alpha \) and \( \gamma \).

The interior solution may not exist under huge transport costs and distances to schools, or under a poorly efficient education sector.

For fixed \( \xi, \mu \) and \( x(i) \), this solution neither exists if the demographic parameters induce markedly low life expectancy and maximal age figures.
Comparative statics for schooling:

**Proposition 2** Under the conditions of Proposition 1, the interior solution $S$ is a strictly increasing function of $\mu$, $\gamma$ and $\alpha$, and a strictly decreasing function of $\beta$, $\xi$ and $x(i)$. 
Is the interior optimal schooling decision dominated by a corner solution?

Possible corner solutions: \( S_t(\mu, i) = 0 \) and \( S_t(\mu, i) = L_t \),

\( S_t(\mu, i) = L_t \) is of course always dominated because of costs of schooling.

Compare the two level of utilities:

\[
\int_{\hat{S}}^{L} \mu \hat{S} e^{\gamma z} m(z) \, dz > \int_{0}^{\hat{S}} \xi x(i) e^{\gamma z} m(z) \, dz + k + e^{-\gamma t} W_0.
\]
Proposition 3  For any fixed cost \( k > 0 \), there exist a threshold \( \tilde{\mu}(i) > \mu(i) \) checking:

(i) If \( \mu > \tilde{\mu}(i) \), then the interior schooling solution is optimal: \( S^* = \hat{S} \).

(ii) If \( \mu < \tilde{\mu}(i) \), then the interior schooling solution is dominated: \( S^* = 0 \).

(iii) If \( \mu = \tilde{\mu}(i) \), then the individual is indifferent between the interior solution and the corner solution.
Proposition 4  For any $t$, the threshold $\tilde{\mu}$ is a strictly increasing function of $k, w^h, \xi, x(i)$ and $\beta$, and a strictly decreasing function of $\alpha$ and $\gamma$. 
Solution to the school location problem

Assume that, provided schools are created, there will be at least one at 0. Hence, the schools are located at \((j - 1)/E\), with \(j = 1, \ldots, E\).

The potential catchment area of the school at 0 is the circular segment \([-1/2E, 1/2E]\).

The distance function \(x(i)\) is the arc length between location \(i\) and the closest school, hence in the catchment area of 0, \(x(i) = i\).

Two cases may occur: separated or contiguous catchment areas
The members of the new-born cohort who attend school have
\[ \mu > \tilde{\mu}(i) \]

\( \tilde{\mu}(\cdot) \) is increasing: for a population very close to the school, many students are likely to attend courses, while for very distant populations, only the most skilled ones will attend.

The attendance rate of population located at \( i \) is given by
\[ \frac{\bar{\mu} - \tilde{\mu}(i)}{\bar{\mu}} \]

Benefit accruing from the pupils at \( i \):
\[ b(i) = k\zeta \left(1 - \frac{\tilde{\mu}(i)}{\bar{\mu}} \right) \]  \hspace{1cm} (12)
Separated catchment areas
with separated catchment areas the benefit is (tuition fee $k$ times the attendance):

$$kR(E) = 2 \int_0^\ell b(i) \, di$$

$$= k \left( 2\ell \zeta - \frac{2\zeta}{\mu} \int_0^\ell \tilde{\mu}(x(i)) \, di \right)$$

$$= k \left( 2\ell \zeta - \bar{M} \right)$$

The global profit when implementing $E$ schools is

$$B(E) = E \left[ k \left( 2\ell \zeta - \bar{M} \right) - f \right]$$

(13)

If $[] > 0$, create as many schools as possible: $\bar{E} = 1/(2\ell)$.
Contiguous catchment areas
Benefit:

\[
kR(E) = 2 \int_0^{2E} b(i) di
\]

\[
= k \left( \frac{\zeta}{E} - \frac{2\zeta}{\tilde{\mu}} \int_0^{2E} \tilde{\mu}(x(i)) di \right)
\]

\[
= k \left( \frac{\zeta}{E} - M(E) \right)
\]

Profit function:

\[
B(E) \equiv k (\zeta - E M(E)) - E f = k \zeta - E [k M(E) + f] \quad (14)
\]

Concave in \(E\). Maximum reached at \(E^* \geq \bar{E}\).
Proposition 5  1. If $2\ell\zeta - \overline{M} - f/k < 0$, then no schools will be created.

2. If $2\ell\zeta - \overline{M} - f/k = 0$, then $\tilde{E}$ is the optimal and equilibrium number of schools.

3. If $2\ell\zeta - \overline{M} - f/k > 0$, then $E^*$ is the optimal number of schools.
Calibration

Calibrate exogenous processes $\alpha_t$, $\beta_t$, $\zeta_t$ and $\gamma_t$ to data.

Polynomials of order 3 in time.
Parameters of the polynomial chosen by minimizing the distance with data.

Choice of the other parameters: $\theta$, $k$, $\bar{\mu}$, $\xi$, $f$, $\omega^h$. 
Mortality process: $m_t(30)$ and $m_t(50)$
Fertility process: $\zeta_t$
Productivity process: $A_t$ and actual TFP
Baseline simulation

Choice of parameters: $\theta = 0.04$, $\bar{\mu} = 1$, $w^h = 1$, $k = 0.05$.

The calibration of $w^h$ will be refined later by computing GDP growth.

No guideline to fix $\xi$ (transportation cost) since distance is arbitrary (normalized by the length of the circle).

$f$ is chosen to have no school in 1530 and some afterwards.
Baseline simulation - school density

1550  1600  1650  1700  1750  1800  1850

0  2  4  6  8
School foundations [Source: Cressy (1980).]
Simulation with constant mortality
Simulation with constant fertility
Little effect from mortality (at least on school creations)

The main part is driven by fertility and population density
Ambiguous role of technical progress

Suppose demographic variables are constant
Technical progress can explain the shift no schools -¿ some schools. (it increases the number potential clients)

But, at higher levels of productivity, it reduces the number of schools: since people really want to go to school, they are ready to walk longer distances.

(result in economic geography: act as a transportation cost: lower transportation cost implies more centralization)
Implications for Literacy and growth

\[
\frac{1}{P_t} \int_{t-L}^{t} \zeta z m_z(t-z) \int_{0}^{1} \frac{\bar{\mu} - \tilde{\mu}_z(i)}{\bar{\mu}} \, di \, dz.
\]

\[
H_t = \int_{t-L}^{t} \zeta z m_z(t-z) \int_{0}^{1} \tilde{\mu}_z(i) \delta(t - z - S_z(\mu, i)) \frac{h_z(\mu, i)}{\bar{\mu}} \, d\mu \, di \, dz.
\]
Problems:

1) Computed literacy too high.

Solution: lognormal distribution of abilities instead of uniform?

2) To compute GDP at one date, it takes 1 night.

Solution:?
Conclusion

Take-off modelled with "population-induced" technical progress

Effect of population derived from primary assumptions

Number and location of education facilities endogenous.

Individuals determine their education investment depending on the distance from the nearest school.

Higher population density makes optimal to multiply the number of schools, opening the possibility to reach higher educational levels.

This effect of population on the number of schools is consistent with the available evidence for England which shows a high rate of school foundations over the period 1550-1650.