The Kinked Demand Curve and Price Rigidity: Evidence from Scanner Data*

Maarten Dossche† Freddy Heylen‡ Dirk Van den Poel§

JOB MARKET PAPER I

Abstract

We estimate the curvature of the demand curve for a wide range of products. We use an extension of Deaton and Muellbauer’s Almost Ideal Demand System and scanner data from a large euro area retailer. Our findings suggest that a sensible parameter value for the curvature of the demand curve, or the price elasticity of the price elasticity, is 4. This implies that the price elasticity of demand is higher for price increases than for price decreases. This value is one to two orders of magnitude smaller than the value economists usually impose. Contrary to what theory suggests, we do not find that items with a different curvature have a different frequency or size of nominal price adjustment in our data.

JEL Codes: C33, D12, E3

Keywords: price setting, real price rigidity, kinked demand, Almost Ideal Demand System

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1 Introduction

A large literature documents the persistent effects of monetary policy on real output and inflation (Christiano et al., 1999, 2005; Peersman, 2004). To match this persistence micro-founded models with sticky prices were developed. A first approach was to introduce frictions to nominal price adjustment (e.g. Taylor, 1980; Calvo, 1983; Mankiw, 1985). However, as shown by several authors, the real effects of nominal frictions do not last much longer than the average duration of a price (Chari et al., 2000; Bergin and Feenstra, 2000). Taking into account recent micro-economic evidence that the mean price duration lies between 1.8 and 4 quarters for the United States (Bils and Klenow, 2004; Nakamura and Steinsson, 2007), and between 4 to 5 quarters for the euro area (Dhyne et al., 2006), nominal frictions alone fail to generate the persistence observed in the data.

The failure of nominal frictions alone to generate persistence has led to the development of models that combine nominal and so-called real price rigidities (Ball and Romer, 1990). Real rigidities refer to strategic complementarity in the price setting decision of firms. A firm is more reluctant to adjust its price in response to changes in the state of the economy the less other firms adjust their prices. Different frictions can generate this strategic complementarity. One option is the roundabout production structure of Basu (1995). Real price rigidity follows from the assumption that firms use the output of all other firms as materials in their own production (Bergin and Feenstra, 2000). A second option is to model firm-specific production factors. In this case the marginal cost is a negative function of the relative price, which again dampens the incentive to change prices.

An alternative, and recently very popular, way to introduce strategic complementarity is the preference specification of Kimball (1995). In contrast to the traditional Dixit and Stiglitz (1977) aggregator, Kimball (1995) no longer assumes a constant elasticity of substitution in

\footnote{E.g. Galí and Gertler, 1999; Sbordone, 2002; Woodford, 2003; Altig et al., 2005; Burstein and Hellwig, 2007.}

demand. The price elasticity of demand becomes a positive function of the relative price. A key concept is the so-called curvature of the demand curve, which measures the price elasticity of the price elasticity. When the curvature is positive, Kimball’s preferences generate a concave or smoothed "kinked" demand curve in a log price/log quantity framework. A price above the level of the firm’s competitors increases the elasticity of demand for its product, so that the firm increasingly loses profits from relative price increases. Conversely, a price below the level of the firm’s competitors reduces the elasticity of demand for its product, so that the firm again increasingly loses profits from relative price decreases. In this way the combination of small costs to nominal price adjustment and a concave demand curve generates slow adjustment to changes in the state of the economy.

Despite its attractiveness, the literature suffers from a lack of empirical evidence on the curvature of a typical demand curve. In Table 1 we report the parameter values for the price elasticity of demand and for the curvature as calibrated or estimated in recent models using macroeconomic data. Values for the (positive) price elasticity range from 3 to 20. Values for the curvature range from less than 2 to more than 400.

Table 1: Price Elasticity and Curvature of Demand in the Literature

<table>
<thead>
<tr>
<th></th>
<th>Price Elasticity</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kimball (1995)</td>
<td>11</td>
<td>471(a)</td>
</tr>
<tr>
<td>Chari, Kehoe and McGrattan (2000)</td>
<td>10</td>
<td>385(a)</td>
</tr>
<tr>
<td>Coenen, Levin and Christoffel (2006)</td>
<td>5 – 20</td>
<td>10, 33</td>
</tr>
<tr>
<td>Smets and Wouters (2007)</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Klenow and Willis (2006)</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Woodford (2005)</td>
<td>7.67</td>
<td>6.67(a)</td>
</tr>
<tr>
<td>Bergin and Feenstra (2000)</td>
<td>3</td>
<td>1.33(a)</td>
</tr>
</tbody>
</table>

Note: Curvature is defined as the elasticity of the price elasticity of demand with respect to the relative price at steady state. Several authors characterize curvature differently. In Appendix 1 we derive the relationships between alternative definitions of curvature. The numbers indicated with (a) have been computed using these relationships. It is sometimes argued that Kimball (1995) would have imposed a curvature equal to 33 (see Eichenbaum and Fisher, 2004; Coenen, Levin and Christoffel, 2006). Our calculations show however that Kimball’s curvature must be much larger.

Our contribution in this paper is twofold. First, we estimate the curvature of the demand curve. To do this, we use scanner data on both prices and quantities from a large euro area
supermarket chain. The dataset contains information about prices and quantities sold of about 15,000 items in 2002-2005. As is typical for studies with micro data, we find wide variation in the estimated curvature of demand among items/product categories. We observe both items with a convex and a concave demand curve. This result would ideally be matched with a model with heterogeneous firms that can match the entire distribution of curvatures. Our results also support the introduction of a kinked (concave) demand curve in a representative firm economy, but the median degree of curvature is much lower than currently calibrated. We suggest to impose a curvature parameter of 4. This finding is consistent with Klenow and Willis (2006) who find that the joint assumption of realistic idiosyncratic shocks and a curvature of 10 is not compatible with observed nominal and relative price changes in US data.

Second, we link the estimates for the curvature of the demand curve to statistics on nominal price adjustment. In case strategic complementarity affects price setting these should be correlated. We find no clear correlation between the estimated curvature and the observed size or frequency of nominal price adjustment in our data. The fact that our data stems from a multi-product retailer may explain this lack of correlation. Midrigan (2006) argues that the item-specific frequency and size of price adjustment in a multi-product firm are also a function of the shocks and price adjustment frictions of the entire product category to which the item belongs. This could explain why the relation between item-specific statistics of price adjustment and the estimated elasticities and curvatures is disturbed in our data.

Section 2 describes the dataset in detail and already analyzes the asymmetry in the price elasticity of demand for price increases versus decreases using an identification with sign restrictions. Section 3 of the paper presents a more rigorous econometric analysis of price elasticities and curvature parameters for individual items. To that end we extend the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) by introducing assumptions drawn from behavioral decision theory. Section 4 concludes the paper.

\footnote{Note that the items that are sold by our retailer can be differently packaged goods of the same brand. All items and/or brands in turn belong to a particular product category (e.g. potatoes, detergent).}
2 Basic Facts about the Data

2.1 Description of Dataset

We use scanner data for a sample of six outlets of an anonymous large euro area supermarket chain. This retailer carries a very broad assortment of about 15,000 different items (stockkeeping units). The products in the total dataset correspond to approximately 40% of the euro area CPI. The data that we use in this paper are prices and total quantities sold per outlet of 2274 individual items belonging to 58 randomly selected product categories. Appendix 2 describes these categories and the number of items in each product category. The time span of our data runs from January 2002 to April 2005. Observations are bi-weekly. Prices are constant during each period of two weeks. They are the same in each of the six outlets. The quantities are the number of packages of an item that are sold during a time period.

2.2 Nominal Price Adjustment

The nominal price friction in our dataset is that prices are predetermined for periods of at least two weeks. If they are changed at the beginning of a period of two weeks, they are not changed again before the beginning of the next period of two weeks, irrespective of demand. A second characteristic of our data is the high frequency of temporary price markdowns. We define the latter as any sequence of three, two or one price(s) that is below both the most left adjacent price and the most right adjacent price. The median item is marked down for 8% of the time, whereas 27% of the median item’s output is sold at times of price markdowns. In line with the previous, price markdowns are valid for an entire period, and not just for a few days.

Using the prices in the dataset, we can estimate the size of price adjustment, the frequency of price adjustment and median price duration as has been done in Bils and Klenow (2004) and Dhyne et al. (2006). Table 2 contains these statistics. The total number of items involved is 2274. Note that due to entry or exit we do not observe data for all items in all periods. We calculate price adjustment statistics including and excluding temporary price markdowns.

4This definition puts us somewhere in between Klenow and Kryvtsov (2005) and Midrigan (2006).
When an observed price is a markdown price, we replace it by the last observed regular price (see also Klenow and Kryvtsov, 2005). We illustrate our procedure in Appendix 3.

Conditional on price changes taking place and including markdowns, we see in Table 2 that 25% of the items have an average absolute price change of less than 5%. At the other end, 25% have an average absolute price change of more than 17%. The median item has an average absolute price change of 9%. Filtering out markdowns, the latter falls to 5%. As to price duration, the median item’s price lasts 0.9 quarters when we include markdown periods. It lasts 6.6 quarters excluding markdown periods. Price duration in our data is only slightly longer than is typically observed in the US and the euro area (Bils and Klenow, 2004; Nakamura and Steinsson, 2007; Dhyne et al., 2006).

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Incl. markdowns</th>
<th>Excl. markdowns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25% 50% 75%</td>
<td>25% 50% 75%</td>
</tr>
<tr>
<td>Average Absolute Size</td>
<td>5%  9% 17%</td>
<td>3%  5%  8%</td>
</tr>
<tr>
<td>Implied Median Price Duration (quarters)</td>
<td>0.4 0.9 2.8</td>
<td>2.4 6.6 ∞</td>
</tr>
</tbody>
</table>

Note: The statistics reported in this table are based on bi-weekly price data for 2274 items belonging to 58 product categories from January 2002 to April 2005. The data show the average absolute percentage price change (conditional on a price change taking place) and the median price duration of the items at the 25th, 50th and 75th percentile, ordered from low to high.

### 2.3 Real Price and Quantity Adjustment

**Relative Importance of Demand and Supply Shocks**

Table 3 presents summary statistics on real (relative) price and quantity changes over the six outlets in our dataset. All changes are again in comparison with the previous period of two weeks. The nominal price \( p_i \) of individual item \( i \) is common across the outlets. All the other data are different per outlet. Real (relative) item prices \( p_i / P^* \) have been calculated by deflating the nominal price of item \( i \) by the outlet-specific Stone price index \( P^* \) for the product category to which the item belongs.\(^5\) The Stone price index is computed as

\[
\ln P^* = \sum_{i=1}^{N} s_i \ln p_i
\]

\(^5\)As an alternative to the Stone index we have also worked with the Fisher index. The results based on this price index are reported in Appendix 4. They confirm our main findings here.
with \( N \) the number of items in the product category to which \( i \) belongs, \( s_i = \frac{p_i q_i}{X} \) the outlet-specific share of item \( i \) in total nominal expenditures \( X \) on the product category, \( q_i \) the total quantity of item \( i \) sold at the outlet and \( X = \sum_{i=1}^{N} p_i q_i \). Total outlet-specific real expenditures \( Q \) on the product category have been obtained as \( Q = X/P^* \). Relative quantities \( q_i/Q \) show much higher and much more variable percentage changes than relative prices. Including markdowns, the average absolute percentage change in relative quantity equals 59\% for the median item, with a standard deviation of 77\%. The average absolute percentage relative price change for the median item equals only 9\%, with a standard deviation of 12\%.

The underlying individual goods data also allow for a first explorative analysis of the importance of supply and demand shocks. To that aim we first calculate simple correlations per item and per outlet between the change in real (relative) item prices and the change in relative quantities sold. In case demand shocks dominate supply shocks, we should mainly find positive correlations between items’ price and quantity changes. In case supply shocks are dominant, we should observe negative correlations. Next we split up the calculated variance in individual items’ real price and quantity changes into a fraction due to supply shocks and a fraction due to demand shocks. The bottom rows of Table 3 show the fractions due to supply shocks. Concentrating on price changes, this fraction has been computed as

\[
\% \text{ Supply shocks to } \Delta \ln(p_i/P^*) = \frac{\sum_{SS} (\Delta \ln(p_i/P^*) - \pi_i)^2}{\sum (\Delta \ln(p_i/P^*) - \pi_i)^2} \times 100
\]

where \( \pi_i \) is the mean of \( \Delta \ln(p_i/P^*) \) over all periods. The numerator of this ratio includes only observations where price and accompanying quantity changes in a period have the opposite sign, revealing a supply shock (SS). The denominator includes all observations. The fraction of the variance in real price changes due to demand shocks, can be calculated as 1 minus the fraction due to supply shocks. Our results reveal that price and quantity changes are mainly driven by supply shocks. Including all data, the median item shows a clearly negative correlation between price and quantity changes equal to -0.23. Moreover, about 65\% of the variance in price and
quantity changes of the median item seems to follow from supply shocks.

The right part of Table 3 presents results obtained from data excluding markdown periods. Temporary price markdowns are interesting supply shocks to identify a possibly kinked demand curve, but we do not consider them as representing idiosyncratic supply shocks such as shifts in costs or technology.\(^6\) As can be seen, the results at the right hand side of the table are fully in line with those at the left hand side.

Table 3: Importance of Demand and Supply Shocks

<table>
<thead>
<tr>
<th></th>
<th>Including markdowns</th>
<th>Excl. markdowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentile</td>
<td>25%</td>
<td>50%</td>
</tr>
<tr>
<td>Average absolute (\Delta \ln(p_i/P^*))</td>
<td>6%</td>
<td>9%</td>
</tr>
<tr>
<td>Average absolute (\Delta \ln(q_i/Q))</td>
<td>39%</td>
<td>59%</td>
</tr>
<tr>
<td>Standard Deviation (\Delta \ln(p_i/P^*))</td>
<td>7%</td>
<td>12%</td>
</tr>
<tr>
<td>Standard Deviation (\Delta \ln(q_i/Q))</td>
<td>52%</td>
<td>77%</td>
</tr>
<tr>
<td>Correlation ((\Delta \ln(p_i/P^*) \cdot \Delta \ln(q_i/Q)))</td>
<td>-0.49</td>
<td>-0.23</td>
</tr>
<tr>
<td>% Supply Shocks to (\Delta \ln(p_i/P^*)) (^{(a)})</td>
<td>48%</td>
<td>68%</td>
</tr>
<tr>
<td>% Supply Shocks to (\Delta \ln(q_i/Q)) (^{(a)})</td>
<td>45%</td>
<td>64%</td>
</tr>
</tbody>
</table>

Note: The statistics reported in this table are based on changes in bi-weekly data for 2274 items belonging to 58 product categories in six outlets. Individual nominal item prices \((p_i)\) are common across the outlets, all the other data \((P^*, q_i, Q)\) can be different per outlet. For the statistical analysis we have excluded items that are mentioned in the supermarket’s circular. Items in the circular are often sold at lower price. Including them may bias the results in favor of supply shock dominance (high quantity sold, low price). For a proper interpretation, note that the median item can be different in each row of this table. \(^{(a)}\) The contribution of demand shocks to price and quantity variability equals 1 minus the contribution of supply shocks. The computation is described in the main text.

An analysis of the relative importance of supply versus demand shocks is important for more than one reason. First, this is important to know in order to do a proper econometric demand analysis. One needs enough variation in supply to be able to identify a demand curve. Our results in Table 3 are obviously encouraging in this respect. The minor contribution of demand shocks should not be surprising given that prices are being set in advance or in the very beginning of the period. As long as the supplier\(^7\) does not know demand in advance, demand shocks cannot have an effect on prices.\(^8\) Second, the results of a decomposition of the

\(^6\)Note that we only exclude the item whose price is marked down, while keeping the other items. The effects of the (excluded) marked down item on the other items are thus not filtered out. If we excluded all items in periods where at least one item in the product category is marked down, we would be left with almost no observations.

\(^7\)When we use the concept 'supplier' we mean the retailer and the producer together. Usually prices in the retail sector are set in an agreement between the retailer and the producer, so that there is not one easily identifiable party that sets prices.

\(^8\)Of course, one could argue that the supplier does know in advance that demand will be high or low, so that
variance of price changes into fractions due to demand and supply shocks may be important for a proper calibration of theoretical macro models. In order to explain large price changes, a number of authors have introduced idiosyncratic shocks in their models, affecting prices and quantities (Golosov and Lucas, 2003; Dotsey, King and Wolman, 2006; Klenow and Willis, 2006; Burstein and Hellwig, 2007). As Klenow and Willis (2006) point out, there is not much empirical evidence available that tells us whether these idiosyncratic shocks are mainly supply-driven or demand-driven. Evidence like ours on the importance of demand and supply shocks excluding markdowns should help to overcome this problem.

**Preliminary Evidence on Asymmetric Price Sensitivity**

An explorative analysis of our data should also provide a first test of the hypothesis that the price elasticity of demand rises in a product’s relative price. Figure 1 illustrates our identification. Per item we relate real (relative) prices to quantities in natural logs. We then demean all relative price and quantity data to account for item specific fixed effects. The average is thus at the origin.

We then want to use supply shocks to identify the demand curve and potential asymmetries he can already at the moment of price setting fix an appropriate price. Considering the large majority of negative correlations in Table 3, however, there is little evidence that this hypothesis would be important.

Figure 1: Identification of Asymmetry in the Demand Curve
in demand. Supply shocks should imply shifts in prices and quantities that go into opposite directions. Our approach to identify the asymmetry in the demand curve is to use only the price-quantity information that is consistent with movements along the bold arrows. In particular, we use all couples of consecutive (log relative) price-quantity observations that lie in the second or fourth quadrant and that have a negative slope. Each couple allows us to calculate a corresponding price elasticity as the inverse of this slope. Price-quantity observations that do not respect this double condition (see the dashed arrows) are not taken into account. Observations of negatively sloped arrows in the first or third quadrant are not considered since it is unclear whether they took place along the (potentially) low or high elasticity part of the demand curve. The last step is to compute the median of all price elasticities that meet our conditions in the second quadrant, where the relative price is high, and to repeat this in the fourth quadrant where the relative price is low.

The data in Table 4 contain the results for the difference between these two median elasticities in absolute value ($\varepsilon^H$ and $\varepsilon^L$). The interpretation of the Table is analogous to earlier tables. The price elasticity of demand at high relative price is higher than at low relative price for most of the items analyzed, which is consistent with the existence of a kinked demand curve. For the median item $\varepsilon^H$ is about 1.3 higher than $\varepsilon^L$. Excluding markdowns hardly affects this result. Note however that a large fraction of items show a convex demand curve. Including markdowns this fraction is 41%, excluding markdowns it is 42%.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Including Markdowns</th>
<th>Excluding Markdowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median $\varepsilon^H - \varepsilon^L$</td>
<td>-3.58</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Note: $\varepsilon^H$ and $\varepsilon^L$ are the absolute values of the price elasticity of demand at high and low relative prices respectively. $\varepsilon^H > \varepsilon^L$ suggests that the demand curve is concave (smoothed "kinked"). Reported data refer to the items at the 25th, 50th and 75th percentile ordered from low to high. Items mentioned in the supermarket’s circular have again been excluded from the analysis (see our note to Table 3).

Our approach here is rudimentary. A more rigorous econometric analysis, which allows us to control for other potential determinants of demand, is necessary. Yet, our results in Table 4 shed
first light on the issue of asymmetry in the elasticity, while imposing only limited conditions on the data and without requiring any functional form assumptions. The evidence is already useful for models like the one of Burstein et al. (2006), where the difference between $\varepsilon^L$ and $\varepsilon^H$ plays a key role in their calibration.\textsuperscript{9} For the other models (e.g. Bergin and Feenstra, 2000; Smets and Wouters, 2007) with a curvature parameter, we need to do a structural analysis.

3 How Large is the Curvature? An Econometric Analysis

In this section we estimate the price elasticity and the curvature of demand for a broad range of goods in our scanner dataset described above. We extend the Almost Ideal Demand System (AIDS) developed by Deaton and Muellbauer (1980) by introducing assumptions drawn from behavioral decision theory. Our "behavioral" AIDS model allows for a more general curvature, which is necessary to answer our research question. The model still has the original AIDS nested as a special case. For several reasons we believe the AIDS is the most appropriate for our purposes: (i) it is flexible with respect to estimating own- and cross-price elasticities; (ii) it is simple, transparent and easy to estimate, allowing us to deal with a large number of product categories; (iii) it is most appropriate in a setup like ours where consumers may buy different items of given product categories; (iv) it is not necessary to specify the characteristics of all goods, and use these in the regressions. The latter three characteristics particularly distinguish the AIDS from alternative approaches like the mixed logit model used by Berry et al. (1995). Their demand model is based on a discrete-choice assumption under which consumers purchase at most one unit of one item of the differentiated product. This assumption is appropriate for large purchases such as cars. In a context where consumers might purchase several items, it may be less suitable. Moreover, to estimate Berry et al. (1995)’s mixed logit model, the characteristics of all goods/items must be specified. In the case of cars this is a much easier task to do than for instance for cement or spaghetti. Computational requirements of their

\textsuperscript{9}In their basic calibration Burstein et al. impose $\varepsilon^H = 9$ and $\varepsilon^L = 3$. Considering our preliminary evidence in this Section the difference between $\varepsilon^H$ and $\varepsilon^L$ seems high.
methodology are also very demanding.

We follow the approach of Broda and Weinstein (2006) to cover as many goods as possible in order to get a reliable estimate for the aggregate curvature. In Section 3.1 we first describe our extension of the AIDS model. Section 3.2 discusses our econometric setup and identification and estimation. Section 3.3 presents the results. In Section 3.4 we discuss their robustness.

3.1 Model

Our extension of Deaton and Muellbauer’s AIDS model is specified in expenditure share form as

\[ s_i = \alpha_i + N \sum_{j=1}^{N} \gamma_{ij} \ln p_j + \beta_i \ln \left( \frac{X}{P} \right) + \sum_{j=1}^{N} \delta_{ij} \left( \ln \left( \frac{p_j}{P} \right) \right)^2 \]  

for \( i = 1, \ldots, N \). In this equation \( X \) is total nominal expenditure on the product category of \( N \) items being analyzed (e.g. detergents), \( P \) is the price index for this product category, \( p_j \) is the price of the \( j \)th item within the product category and \( s_i \) is the share of total expenditures allocated to item \( i \) (i.e. \( s_i = p_i q_i / X \)). Deaton and Muellbauer define the price index \( P \) as

\[ \ln P = \alpha_0 + N \sum_{j=1}^{N} \alpha_j \ln p_j + \frac{1}{2} \sum_{j=1}^{N} \sum_{i=1}^{N} \gamma_{ij} \ln p_i \ln p_j \]  

Our extension of the model concerns the last term at the right hand side of Equation (2). The original AIDS model has \( \delta_{ij} = 0 \). Although this model is generally recognized to be flexible, it is not flexible enough for our purpose. As we demonstrate below, the curvature parameter is not free in the original AIDS model. It is a restrictive function of the price elasticity, implying that in the original AIDS model it would not be possible to obtain a convex demand curve.

In extending the AIDS model we are inspired by relatively recent contributions to the theory of consumer choice, which draw on behavioral decision theory and imply asymmetric consumer reactions to price changes. Seminal work on loss aversion was done by Kahneman, Tversky and Thaler. An important idea in these contributions is that consumers evaluate choice alternatives not only in absolute terms, but as deviations from a reference point (e.g. Tversky and Kahneman, 1991; Thaler, 1985). A popular representation of this idea is that consumers form
a reference price, with deviations between the actual price and the reference price conveying utility, and thus influencing consumer purchasing behavior for a given budget constraint (see Putler, 1992). We translate this idea to the context of standard macro models where consumers base their decisions on the price of individual goods relative to the aggregate price, as in Dixit and Stiglitz (1977) or Kimball (1995). The aggregate price would thus be the reference price. Within this broader approach, consumers will not only buy less of a good when its price rises above the aggregate price due to standard substitution and income effects, but also because the relative price rise creates an additional loss. The consumer feels being treated unfairly, like in Okun (1981) or Rotemberg (2002). Inversely, the appetite for a good increases when its price decreases below the aggregate price.

Figure 2 illustrates this argument. A key element is that the slope of an indifference curve through a single point in a two-good space will depend on whether the actual price is relatively high or low compared to the relevant aggregate (reference) price. Initial prices of goods 1 and 2 are $p^a_1$ and $p^a_2$. Both are equal to the aggregate price. The consumer maximizes utility when she buys $q^a_1$ (point a). Then assume a price increase for good 1 to $p^b_1$, rotating the budget line downwards. Traditional income and substitution effects will make the consumer move to point b, reducing the quantity of good 1 to $q^b_1$. Additional relative (or reference) price effects, however, will now shift the indifference surface. With $p_1$ now relatively high, the indifference curve through point b will become flatter. Since buying good 1 conveys utility losses, the consumer is willing to give up less of good 2 for more of good 1. The consumer reaches a new optimum at point d. Relative price effects on utility therefore induce an additional drop in $q_1$ to $q^d_1$. A similar graphical experiment can be done for a fall in $p_1$. Tversky and Kahneman’s (1991) loss aversion hypothesis would then predict opposite, but smaller relative price effects, implying a kink in the demand curve (see also Putler, 1992).

The implication of this argument is that relative price effects on the indifference surface
should be accounted for in demand analysis. The added term \( \sum_{j=1}^{N} \delta_{ij} \left( \ln \left( \frac{p_j}{P} \right) \right)^2 \) in Equation (2) allows us to capture these additional effects. Provided that standard adding up (\( \sum_{i=1}^{N} \alpha_i = 1, \sum_{i=1}^{N} \gamma_{ij} = 0, \sum_{i=1}^{N} \beta_i = 0, \sum_{i=1}^{N} \delta_{ij} = 0 \)), homogeneity (\( \sum_{j=1}^{N} \gamma_{ij} = 0 \)) and symmetry (\( \gamma_{ij} = \gamma_{ji} \)) restrictions hold, our extended equation is a valid representation of preferences.

Figure 2. The Effects of Increasing the Price of Good 1

The definition of the (positive) uncompensated own price elasticity of demand for good \( i \) is:

\[
\varepsilon_i = \frac{\partial \ln q_i}{\partial \ln p_i} = 1 - \frac{\partial \ln s_i}{\partial \ln p_i}
\]

where \( q_i = s_i X / p_i \). Applied to our behavioral AIDS model, \( \varepsilon_i \) can then be derived from Equation (2) as

\[
\varepsilon_{i(B-AIDS)} = 1 - \frac{1}{s_i} \left( \gamma_{ii} - \beta_i \frac{\partial \ln P}{\partial \ln p_i} + 2 \delta_{ii} \ln \left( \frac{p_i}{P} \right) - 2 \sum_{j=1}^{N} \delta_{ij} \ln \left( \frac{p_j}{P} \right) \frac{\partial \ln P}{\partial \ln p_i} \right)
\]

where we hold total nominal expenditure on the product category \( X \) as well as all other prices \( p_j (j \neq i) \) constant. In the AIDS model the correct expression for the elasticity of the group
price $P$ with respect to $p_i$ is
\[
\frac{\partial \ln P}{\partial \ln p_i} = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_j \tag{6}
\]
However, since using the price index from Equation (3) often raises empirical difficulties (see e.g. Buse, 1994), researchers commonly use Stone’s geometric price index $P^*$, given by (1). The model is then called the "linear approximate AIDS" (LA/AIDS). To obtain the own price elasticity for the LA/AIDS model, one has to start from Stone’s $P^*$ and derive
\[
\frac{\partial \ln P^*}{\partial \ln p_i} = s_i + \sum_{j=1}^{N} s_j \ln p_j \frac{\partial \ln s_j}{\partial \ln p_i} \tag{7}
\]
Green and Alston (1990) and Buse (1994) discuss several approaches to computing the LA/AIDS price elasticities depending on the assumptions made with regard to $\frac{\partial \ln s_j}{\partial \ln p_i}$ and therefore $\frac{\partial \ln P^*}{\partial \ln p_i}$. A common approach is to assume $\frac{\partial \ln s_j}{\partial \ln p_i} = 0$, such that $\frac{\partial \ln P^*}{\partial \ln p_i} = s_i$. Monte Carlo simulations by Alston et al. (1994) and Buse (1994) reveal that this approximation is superior to many others (e.g. smaller estimation bias). In our empirical work we will also use Stone’s price index and this approximation. The (positive) uncompensated own price elasticity implied by this approach then is
\[
\varepsilon_i^{(LA/AIDS)} = 1 - \frac{\gamma_{ii}}{s_i} + \frac{\beta_i}{s_i} - \frac{2 \delta_{ii} \ln \left( \frac{p_i}{P^*} \right)}{s_i} + 2 \sum_{j=1}^{N} \delta_{ij} \ln \left( \frac{p_j}{P^*} \right) \tag{8}
\]
Equation (8) incorporates several channels for the relative price of an item to affect the price elasticity of demand. The contribution of our behavioral extension of the AIDS model is obvious from the presence of $\delta_{ii}$ in this equation. Since $s_i$ is typically far below 1, observing $\delta_{ii} < 0$ will most likely imply a concave demand curve, with $\varepsilon_i$ rising in the relative price $\frac{p_i}{P^*}$. When $\delta_{ii} > 0$, it is more likely to find convexity in the demand curve.

At steady state, for all relative prices equal to 1, the price elasticity becomes
\[
\varepsilon_i^{(LA/AIDS)(1)} = 1 - \frac{\gamma_{ii}}{s_i} + \beta_i \tag{9}
\]
Finally, starting from Equation (8) we show in Appendix 5 that the implied curvature of
the demand function at steady state is

$$
\varepsilon_{i(LA/B-AIDS)} = \frac{\partial \ln \varepsilon_i}{\partial \ln p_i} = \frac{1}{\varepsilon_i} \left((\varepsilon_i - 1) (\varepsilon_i - 1 - \beta_i) - \frac{2\delta_{ii}(1-s_i)}{s_i} + 2(\delta_{ii} - s_i \sum_{j=1}^{N} \delta_{ij})\right)
$$

Also in this equation the role of $\delta_{ii}$ is clear. For given price elasticity, the lower $\delta_{ii}$, the higher the estimated curvature.

A simple comparison of the above results with the price elasticity and the curvature in the basic LA/AIDS model underscores the importance of our extension. Putting $\delta_{ii} = \delta_{ij} = 0$, one can derive for the basic LA/AIDS model that

$$
\varepsilon_{i(LA/AIDS)} = 1 - \frac{\gamma_{ii}}{s_i} + \beta_i
$$

$$
\varepsilon_{i(LA/AIDS)} = \frac{(\varepsilon_i - 1)(\varepsilon_i - 1 - \beta_i)}{\varepsilon_i}
$$

With $\beta_i$ mostly close to zero (and zero on average) the curvature then becomes a restrictive and rising function of the price elasticity, at least for $\varepsilon_i > 1$. Moreover, positive price elasticities $\varepsilon_i$ almost unavoidably imply positive curvatures, which excludes convex demand curves. In light of our findings in Table 4 this is too restrictive.

### 3.2 Identification/Estimation

The sample that we use for estimation contains data for 28 product categories sold in each of the six outlets (supermarkets). The time frequency is a period of two weeks, with the time series running from the first bi-week of 2002 until the 8th bi-week of 2005. The selection of the 28 categories, coming from 58 in Section 2, is driven by data requirements and motivated in Appendix 2.

To keep estimation manageable we include five items per product category. Four of these items have been selected on the basis of clear criteria to improve data quality and make estimation possible. The fifth item is called "other". It is constructed as a weighted average of all other items. We include "other" to fully capture substitution possibilities for the four main
items. Specifying "other" also enables us to deal with entry and exit of individual items during the sample period.\textsuperscript{10} We discuss the selection of the four items and the construction of "other" in Appendix 2 as well. For each item $i$ within a product category the basic empirical demand specification is:

$$ s_{imt} = \alpha_{im} + \sum_{j=1}^{5} \gamma_{ij} \ln p_{jt} + \beta_i \ln \left( \frac{X_{mt}}{P_{mt}^*} \right) + \sum_{j=1}^{5} \delta_{ij} \left( \ln \left( \frac{P_{jt}}{P_{mt}^*} \right) \right)^2 + \sum_{j=1}^{5} \varphi_{ij} C_{jt} + \lambda_{it} + \varepsilon_{imt} $$

for $i = 1, \ldots, 5$ and $m = 1, \ldots, 6$ and $t = 1, \ldots, 86$ (14)

where $s_{imt}$ is the share of item $i$ in total product category expenditure at outlet $m$ and time $t$, $X_{mt}$ is overall product category expenditure at outlet $m$ and time $t$, $P_{mt}^*$ is Stone's price index for the category at outlet $m$ and $p_{jt}$ is the price of the $j$th item in the category. As we mentioned before, individual item prices are equal across outlets and predetermined. They are not changed during the period. This is an important characteristic of our data, which strongly facilitates identification of the demand curve (cf. infra). Furthermore, $\alpha_{im}$ captures item specific and outlet specific fixed effects.\textsuperscript{11} Finally, we include dummies to capture demand shocks with respect to item $i$ at time $t$ which are common across outlets. Circular dummies $C_{jt}$ are equal to 1 when an item $j$ in the product category to which $i$ belongs, is mentioned in the supermarket’s circular. The circular is common to all outlets. For each item we also include three time dummies $\lambda_{it}$ for New Year, Easter and Christmas. These dummies should capture shifts in market share from one item to another during the respective periods.

Our estimation method is SUR. The assumption underlying this choice is that prices $p_{it}$ are uncorrelated with the error term $\varepsilon_{imt}$. For at least two reasons we believe this assumption is justified. Problems to identify the demand curve, as discussed by e.g. Hausman et al. (1994), Hausman (1997) and Menezes-Filho (2005), should therefore not exist. First, since our retailer sets prices in advance and does not change them to equilibrate supply and demand in a given

\textsuperscript{10}The specification of "other" may also come at a cost. Including "other" imposes a number of restrictions on the regression. In Section 3.4. we briefly reconsider this issue.

\textsuperscript{11}To control for item specific fixed effects, note that we have also de-meaned $\ln \left( \frac{P_{jt}}{P_{mt}^*} \right)$ when introducing the additional term $\sum_{j=1}^{5} \delta_{ij} \left( \ln \left( \frac{P_{jt}}{P_{mt}^*} \right) \right)^2$ in the regression.
period, prices can be considered predetermined with respect to Equation (14). Second, prices are equal in all six outlets. We assume that outlet specific demand shocks for an item do not affect the price of that item at the chain level. Of course, against these explanations one could argue that the supplier may know in advance that demand will be high or low, so that he can already at the moment of price setting fix an appropriate price. However, our results in Section 2.3. do not provide strong evidence for this hypothesis. Demand shocks are of relatively minor importance in driving price and quantity changes. Moreover, most demand shocks should be captured by the circular dummies ($C_{jt}$) and the item specific time dummies ($\lambda_{it}$) in our regressions. They will not show up in the error term. In the same vein, the included fixed effect $\alpha_{im}$ captures the influence on expenditure shares of time-invariant product specific characteristics which will also affect the price charged by the retailer. Therefore, item specific characteristics will not show up in the error term of the regressions either. A robustness test we discuss in Section 3.4. provides additional support for our assumption that prices $p_{it}$ are uncorrelated with the error term $\varepsilon_{imt}$. Using IV methods we obtain very similar results as the ones reported below.

Following Hausman et al. (1994) we estimate Equation (14) imposing homogeneity and symmetry from the outset (i.e. $\sum_{j=1}^{5} \gamma_{ij} = 0$ and $\gamma_{ij} = \gamma_{ji}$). We also impose symmetry on the effects of the circular dummies (i.e. $\varphi_{ij} = \varphi_{ji}$). Finally, the adding up conditions ($\sum_{i=1}^{5} \alpha_{im} = 1$, $\sum_{i=1}^{5} \gamma_{ij} = 0$, $\sum_{i=1}^{5} \beta_{i} = 0$, $\sum_{i=1}^{5} \delta_{ij} = 0$, $\sum_{i=1}^{5} \varphi_{ij} = 0$) allow us to drop one equation from the system. We drop the equation for "other".

3.3 Results

Estimation of Equation (14) for 28 product categories over six outlets, with each product category containing four items, generates 672 estimated elasticities and curvatures. Since 6 of these

---

12Hausman et al. (1994) and Hausman (1997) make a similar assumption. See our brief discussion in Section 3.4.
elasticities were implausible, we decided to drop them, leaving 666 plausible estimates.\footnote{These 6 price elasticities were lower than -10 (where our definition is such that the elasticity for a negatively sloped demand curve should be a positive number). Note that we do not include the estimated elasticities and curvatures for the composite "other" item in our further discussion. Due to the continuously changing composition of this "other" item over time, any interpretation of the estimates would be difficult.}

First, as we cannot discuss explicitly the 666 estimated elasticities and curvatures, we present our results in the form of a histogram in Figures 3 and 4. We find that the unweighted median price elasticity is 1.4. The unweighted median curvature is 0.8. If we weight our results with the turnover each item generates, we do not find very different results. We find a median weighted elasticity of 1.2 and a median weighted curvature of 0.8. Considering the values that general equilibrium modelers impose when calibrating their models, these are low numbers (see Table 1). The elasticities that we find are also a bit low in comparison with the existing empirical literature (see Bijmolt et al., 2005). The main reason for our relatively low price elasticity seems to be the overrepresentation of necessities (e.g. cornflakes, baking flour, mineral water) in the product categories that we could draw from our dataset. The estimated price elasticities for luxury goods, durables and large-ticket items (e.g. smoked salmon, wine, airing cupboards) are generally higher.

Figure 5 and Table 5 bring more structure in our estimation results. Excluding some extreme values for the curvature, Figure 5 reveals that the estimated price elasticity and curvature are strongly positively correlated. The correlation coefficient is 0.53.\footnote{Figure 5 excludes 38 observations with an estimated curvature higher than 40 or lower than -40. If we exclude only observations with a curvature above +60 or below -60, the correlation is 0.51. Note that most of the extreme estimates for the curvature occur when the estimated price elasticity is very close to zero. Relatively small changes in the absolute value of the elasticity then result into huge percentage changes in the elasticity and, according to our definition, high curvature.} In Table 5 we report the unweighted median elasticity and curvature, and their correlation, conditional on the elasticity taking certain values. The condition that the elasticity is strictly higher than 1 corresponds to the approach in standard macroeconomic models. When we impose this condition, the median estimated price elasticity is 2.4, the median estimated curvature 1.7. Imposing that the elasticity is strictly higher than 3 further raises the median curvature to 5.7. Estimated price elasticities between 3 and 6 go together with a median curvature of 3.5.
We can now reduce the uncertainty around the curvature parameter in calibrated macro models. The empirical literature on the price elasticity of demand surveyed by Bijmolt et al. (2005) reveals a median elasticity of about 2.2. Only 9% of estimated elasticities exceed 5. More or less in line with these results, the recent industrial organization literature reports price-cost mark-ups that are consistent with price elasticities between 3 and 6 (see e.g. Domowitz et al., 1988; Konings et al., 2001; Dobbelaere, 2004). Combining these results with our findings in Table 5, a sensible value to choose for the curvature would be around 4. Note that this value is fairly robust to changes in our selection of product categories. Our interpretation of Figure 5 and Table 5 allows us to overcome the bias on our median estimates that may result from an overrepresentation of low-elasticity product categories. Clearly, a value for the curvature of 4 is far below current practice (see again Table 1). Only Bergin and Feenstra (2000) impose a lower value.

Figure 3: Estimation Results Elasticity
Second, our estimated curvatures show that the constant elasticity Dixit-Stiglitz (1977) benchmark is too simplistic. Over the broad range of product categories that we have studied, convex and concave demand curves coexist. Fully in line with our results in Table 4, we observe a negative curvature for 42% of the items. About 27% of our estimated curvatures are below
-2, 38% are above +2. The high frequency of non-zero estimated curvatures, including many negative curvatures, supports our argument that the original AIDS model is too restrictive to answer our research question. A key parameter in our behavioral extension is $\delta_{ii}$ (see our discussion of Equation (8)). Additional tests show that this extension makes sense. We find the estimated $\delta_{ii}$ to be statistically different from zero at the 10% significance level for 43% of the items. Furthermore, a Wald test rejects the null hypothesis that $\delta_{11} = \delta_{22} = \delta_{33} = \delta_{44} = 0$ at the 5% significance level for two thirds of the included product categories. Appendix 6 provides details. A macroeconomic model that fits the microeconomic evidence well should thus ideally allow for sectors with differing elasticities and curvatures.

Third, in order to find out whether a concave demand curve gives rise to stickier prices, we check whether there is a link between our results on the curvature/elasticity and the size/frequency of price adjustment. In other words, does the supplier act differently for products with a high curvature compared to products with a low curvature, as the theory on strategic complementarity would suggest. We computed the correlation between the statistics on nominal price adjustment presented in Table 2 with the 666 estimated elasticities and curvatures. Table 6 reports the results. Only the correlation between the price elasticity of demand and the size of price adjustment is significantly negative for both the cases including and excluding mark-downs. This result provides some evidence in favor of the role of competition and firm-specific production factors to generate real price rigidity. Our estimated curvatures are not correlated with either the frequency or the size of price adjustment. This finding applies irrespective of including or excluding mark-downs. It also applies irrespective of any condition on the level of the curvature (e.g. $\epsilon > 0$) or the elasticity (e.g. $\varepsilon > 1$). This may cast doubt on whether the curvature of the demand curve is really an additional source of price rigidity. However, an issue that might drive the absent correlation between the curvature and the frequency and size of price adjustment is the fact that our data refer to a multi-product firm. Midrigan (2006) documents that multi-product stores tend to adjust prices of goods in narrow product categories.
simultaneously. Price adjustment of individual items in a multi-product firm is also a function of the shocks and price adjustment frictions of the entire product category to which the item belongs. This kind of coordination will break the potential relation between individual items’ curvatures and frequency and size of price adjustment.

Table 6: Correlation with Nominal Price Adjustment Statistics

<table>
<thead>
<tr>
<th></th>
<th>Including Markdowns</th>
<th>Excluding Markdowns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Size</td>
</tr>
<tr>
<td>Elasticity</td>
<td>0.04</td>
<td>-0.09*</td>
</tr>
<tr>
<td>Curvature</td>
<td>0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: The correlations in this Table are calculated using the 666 item elasticity/curvature estimates and their corresponding size and frequency of price adjustment. An asterisk signals that the correlation coefficient is significant at the 5% significance level. The column “Excluding Markdowns” indicates that the size and frequency of price adjustment were calculated discarding periods of temporary price markdowns.

3.4 Robustness

We test the robustness of our results in various ways. First, we have changed the estimation methodology. The assumption underlying the use of SUR is that prices \( p_{it} \) in Equation (14) are uncorrelated to the error term \( \varepsilon_{imt} \). Although we believe we have good reasons to make this assumption, we drop it as a robustness check, and re-estimate our model using an IV method. Ideally, one can use information on costs, e.g. material prices, as instruments. However, data on a sufficient number of input prices with a high enough frequency is generally not available. Hausman et al. (1994) and Hausman (1997), who also use prices and quantities in different outlets, solve this problem by exploiting the panel structure of their data. They make the identifying assumption that prices in all outlets are driven by common cost changes which are themselves independent of outlet specific variables. Demand shocks that may affect the price of an item in one outlet are assumed not to affect the price of that item in other outlets. Prices in other outlets then provide reliable instruments for the price in a specific outlet. This procedure cannot work in our setup since prices are identical across outlets. As an alternative we use once to three times lagged prices \( p_i \) and once lagged relative prices \( \frac{p_i}{p_{i-1}} \) as instruments. Re-estimating
our model for a large subset of the included product categories with the 3SLS methodology, we
generate very similar results for the elasticities and curvatures.

As a second robustness check we introduce seasonal dummies to capture possible demand
shifts related to the time of the year. As we mentioned before, when suppliers are aware of such
demand shifts they may fix their price differently. Not accounting for these demand shifts may
then introduce correlation between the price and the error term, and undermine the quality of
our estimates. Re-estimating our model with additional seasonal dummies does not affect our
results in any serious way either.

Third, we allow for gradual demand adjustment to price changes by adding a lagged depen-
dent variable to the regression. Although often statistically significant, we generally find the
estimated parameter on this lagged dependent variable to be between +0.1 and -0.1. Gradual
adjustment seems to be no important issue in our dataset.

Fourth, our results are based on the assumption that the aggregate price \( P_t^* \) is the relevant
reference price when consumers make their choice. This assumption is in line with the approach
in standard macro models. In the marketing literature, however, it is often assumed that
reference prices are given at the time of choice (see e.g. Putler, 1992; Bell and Latin, 2000). As
a fourth robustness test we have therefore assumed the reference price to be equal to the one-
period lagged aggregate price \( P_{t-1}^* \). Re-estimating our model for a subset of product categories
we find that this alternative has no influence on the estimated price elasticities. It implies slightly
higher estimated curvatures for most items, however without affecting any of our conclusions
drawn above\(^\text{15}\).

A final check on the reliability of our results considers potential implications of the way we
specify and introduce "other". Although necessary to make estimation manageable, introducing
"other" imposes a large number of restrictions on the regression. In Appendix 7 we report
additional statistics showing that there is no correlation at all between the market share of

\(^{15}\) Assuming that the reference price equals \( P_{t-1}^* \) affects the equation for the curvature. Instead of Equation
(11) it then holds that

\[
\epsilon_i = \frac{\partial \ln \pi_i}{\partial \ln p_i} = \frac{(\epsilon_{i-1} - (1 - \beta_i) - 2\theta_{i-1}/\epsilon_i)}{\epsilon_i}.
\]
"other" in a product category and the average estimated elasticity and curvature for the four items in that product category. The estimated elasticity and curvature are not correlated either with the total number of items in the category.

4 Conclusion

The failure of nominal frictions to generate persistent effects of monetary policy shocks has led to the development of models that combine nominal and real price rigidities. Many researchers recently introduce a kinked (concave) demand curve as an attractive way to obtain real price rigidities. However, the literature suffers from a lack of empirical evidence on the extent of curvature in the demand curve. This paper uses scanner data from a large euro area supermarket chain to estimate the curvature of a large number of items.

First, as is typical for studies with micro data, we find wide variation in the estimated curvature of demand among items/product categories. We observe both items with a convex and a concave demand curve. This result would ideally be matched with a model with heterogeneous firms that can match the entire distribution of curvatures. Our results also support the introduction of a kinked (concave) demand curve in a representative firm economy, but the median degree of curvature is much lower than currently calibrated. We suggest to impose a curvature parameter of 4. This finding is consistent with Klenow and Willis (2006) who find that the joint assumption of realistic idiosyncratic shocks and a curvature of 10 is not compatible with observed nominal and relative price changes in US data.

Second, exploiting the heterogeneity in the estimates, we do not find that items with a different curvature have a different frequency or size of nominal price adjustment in our data. In case strategic complementarity affects price setting these should be correlated. The fact that our data stems from a multi-product retailer may explain this lack of correlation. Midrigan (2006) argues that the item-specific frequency and size of price adjustment in a multi-product firm are also a function of the shocks and price adjustment frictions of the entire product.
category to which the item belongs. This could explain why the relation between item-specific statistics of price adjustment and the estimated elasticities and curvatures is disturbed in our data.
References


[37] Midrigan V., 2006, Menu Costs, Multi-Product Firms, and Aggregate Fluctuations, Ohio State University, *mimeo*


Appendix 1: Different Curvatures

Curvature is not defined homogeneously across the different papers in the literature on price rigidity. In this appendix we derive the relationships between the alternative definitions. These relationships underly some of the parameter values that we report in Table 1 in the main text.

We use the following notation: $x_i = q_i/Q$ is firm $i$’s relative output, $p_i$ is its price, $\varepsilon(x_i)$ is the (positive) price elasticity of demand, $\mu(x_i) = \frac{\varepsilon(x_i)}{\varepsilon(x_i) - 1}$ is the firm’s desired markup. Assuming an aggregate price level equal to 1, $p_i$ also indicates the firm’s relative price.

Eichenbaum and Fisher (2004) and Smets and Wouters (2007) define curvature as we have done as the elasticity of the price elasticity of demand with respect to the relative price at steady state:

$$
\epsilon = \left[ \frac{\partial \varepsilon(x_i)}{\partial p_i} \frac{p_i}{\varepsilon(x_i)} \right]_{x_i=1}
$$

Coenen, Levin and Christoffel (2006) define the curvature of the demand curve as the relative slope of the price elasticity of demand around steady state:

$$
\epsilon = \left[ - \frac{\partial \varepsilon(x_i)}{\partial x_i} \right]_{x_i=1}
$$

It can be shown that in steady state both approaches are identical:

$$
\frac{\partial \varepsilon(x_i)}{\partial p_i} \frac{p_i}{\varepsilon(x_i)} = \frac{\partial \varepsilon(x_i)}{\partial x_i} \frac{p_i}{\varepsilon(x_i)} \frac{\partial x_i}{\partial x_i} \frac{x_i}{x_i} = \frac{\partial \varepsilon(x_i)}{\partial x_i} \frac{p_i \partial x_i}{\varepsilon(x_i)} \frac{x_i}{x_i} \frac{p_i \varepsilon(x_i)}{\varepsilon(x_i)} = - \frac{\partial \varepsilon(x_i)}{\partial x_i} \frac{x_i}{\varepsilon(x_i)}
$$

Evaluated at steady state ($x_i = 1$), this is equal to $- \frac{\partial \varepsilon(x_i)}{\partial x_i}$.

Kimball (1995) and Woodford (2005) characterize the curvature in the demand curve by the elasticity of the firm’s desired markup with respect to relative output at steady state, i.e.

$$
\xi = \left[ \frac{\partial \mu(x_i)}{\partial x_i} \frac{x_i}{\mu(x_i)} \right]_{x_i=1}
$$

The relationship between $\epsilon$ and $\xi$ is as follows:

$$
\xi = \left[ \frac{\partial \mu(x_i)}{\partial x_i} \frac{x_i}{\mu(x_i)} \right]_{x_i=1} = \left[ \frac{\partial \mu(x_i)}{\partial x_i} \frac{p_i \varepsilon(x_i)}{\mu(x_i)} \right]_{x_i=1} = \left[ \frac{\partial \varepsilon(x_i)}{\partial x_i} \frac{1}{\varepsilon(x_i)} (\varepsilon(x_i) - 1) \right]_{x_i=1} = \frac{\epsilon}{(\varepsilon(1) - 1) \varepsilon(1)}
$$
Kimball (1995) assumes $\xi = 4.28$ and $\varepsilon(1) = 11$. Woodford imposes $\xi = 0.13$ and $\varepsilon(1) = 7.67$.

The approach in Chari et al. (2000) is very close to Eichenbaum and Fisher (2004), Coenen, Levin and Christoffel (2006) and Smets and Wouters (2007). Cost minimization by households buying differentiated products $i$ to achieve optimal composite consumption $Q$ yields the following first order condition for demand:

$$ p_i = \frac{\lambda}{Q} G'(x_i) $$

with $\lambda$ the Lagrangian lambda on the constraint relating household composite consumption $Q$ to individual quantities $q_i$, $G$ the Kimball (1995) aggregator function for composite consumption and (as defined before) $x_i = q_i/Q$. Rewriting this first order condition, we obtain the demand curve $x_i = D(p_i Q/\lambda)$ with $D = (G')^{-1}$. The price elasticity of demand equals

$$ \varepsilon(x_i) = -\frac{D'(G'(x_i))G'(x_i)}{x_i} $$

Evaluated at steady state this is $\varepsilon(1) = -D'(G'(1))G'(1)$. The curvature of the demand curve at steady state can then be obtained as:

$$ \epsilon = \left[ -\frac{\partial \varepsilon(x_i)}{\partial x_i} \right]_{x_i=1} = D''(G'(1))G''(1)(1) + G''(1)D'(G'(1)) - D'(G'(1))G'(1) $$

Since $D'(G'(1)) = 1/G''(1)$ it follows that

$$ \epsilon = \frac{D''(G'(1))G'(1)}{D'(G'(1))} + 1 + \varepsilon(1) $$

Chari et al. (2000) define their curvature parameter $\chi$ as

$$ \chi = -\frac{D''(G'(1))G'(1)}{D'(G'(1))}, \quad (18) $$

from which the relationship with $\epsilon$ is:

$$ \epsilon = -\chi + 1 + \varepsilon(1) \quad (19) $$

Chari et al. (2000) state a value of -289 for $\chi$ and 10 for $\varepsilon(1)$. According to Equation (19) this would imply $\epsilon = 300$. The discrepancy with the value of 385 that we report in Table 1 is
due to the fact that Chari et al. (2000) use a first order Taylor series expansion of the demand elasticity around the steady state to calculate their curvature parameter $\chi$ associated with the Kimball (1995) parameterisation. The exact value of $\chi$ would be -374.

Finally, Bergin and Feenstra (2000) derive a concave demand curve from assuming preferences with a translog functional form. The (positive) own price elasticity of demand is $\varepsilon_i = 1 - \frac{2\gamma_{ii}}{s_i}$ with $s_i$ the expenditure share of good $i$ and $\gamma_{ii} = \partial s_i / \partial \ln p_i < 0$. Along the lines set out in Section 3.1. of this paper it can be derived that $\epsilon = \frac{(1+\varepsilon_i)^2}{\varepsilon_i}$ . Starting from the imposed $\varepsilon(1) = 3$, $\epsilon$ should be 1.33.
Appendix 2: Description of Dataset

Table 7 gives an overview of the 58 product categories that are in the dataset that we use in this paper. Between brackets we indicate the number of items within each category. The available data for all these categories have been used to compute the basic statistics in Section 2. Product categories in italic are also included in the econometric analysis in Section 3.

Table 7: Product Categories and Number of Items

<table>
<thead>
<tr>
<th>Category</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drinks</td>
<td>tea (67), coke (39), chocolate milk (9), lemonade (33), mineral water (66), wine (17) port wine (54), gin (21), fruit juice (54), beer (6), whiskey (82)</td>
</tr>
<tr>
<td>Food</td>
<td>corn flakes (49), tuna (46), smoked salmon (18), biscuit (9), mayonnaise (45), tomato soup (5), Emmental cheese (56), gruyere cheese (19), spinach (29), margarine (62), potatoes (26), liver torta (98), baking flour (18), spaghetti (30), coffee biscuits (5), minarine (2)</td>
</tr>
<tr>
<td>Equipment</td>
<td>airing cupboard (61), knife (19), hedge shears (32), dishwasher (43), washing machine (36), tape measure (15), tap (24), dvd recorder (20), casserole (74), toaster (40)</td>
</tr>
<tr>
<td>Clothes and related</td>
<td>jeans (79), jacket (88)</td>
</tr>
<tr>
<td>Cleaning products</td>
<td>dishwasher detergent (43), detergent (43), soap powder (98), floorcloth (11) toilet soap (34)</td>
</tr>
<tr>
<td>Leisure and education</td>
<td>hometrainer (52), football (32), cartoon (86), dictionary (32), school book (34)</td>
</tr>
<tr>
<td>Personal care</td>
<td>plaster (33), nail polish (15), handkerchief (63), nappy (64), toilet paper (13)</td>
</tr>
<tr>
<td>Other</td>
<td>potting soil (33), cement (43), bath mat (48), aluminium foil (5)</td>
</tr>
</tbody>
</table>

Note: The number of items in a particular product category is stated in brackets. Only the product categories in italic are included in the econometric analysis in Section 3.

Our econometric analysis in Section 3 includes four items per product category and a composite of all other items in the category, called "other". Including more than four items could make sense from the perspective of covering a larger share of the market. However, it would also imply an inflation of coefficients to be estimated. Moreover, since the price of each item occurs as an explanatory variable in the expenditure share equation of all included items within the product category, raising the number of items could limit estimation capacity when additional items have shorter or non-overlapping data availability. Our criteria to select the four items per product category reflect these concerns. These criteria are (long) data availability and (relatively high) market share within the category.\footnote{Note that both these criteria are strongly (positively) correlated.} More precisely, we ranked all items within the category on the basis of the total number of observations available (the maximum being 86), and chose those items with the highest number of observations. Among items with an
equal number of observations we selected those with the highest market share. If this procedure implied different selections among the six available outlets, we chose those products with the best ranking in most outlets.

The market share of "other" has been constructed as

$$ s_{other} = \frac{X_{other}}{X} = \frac{\sum_{j \in S4} p_j q_j}{X} $$

with $S4$ the selected four items, and all other variables as defined in the main text. The price index of "other" is the Stone index for all items included in "other".

$$ p_{other} = \sum_{j \notin S4} \frac{s_j p_j}{X_{other}} $$

with $s_j = p_j q_j / X_{other}$. Due to different weights $p_{other}$ will differ across the six outlets.

The reduction to 28 product categories in the econometric analysis in Section 3, coming from 58, has been driven by the following criteria. For a category to be included in the econometric analysis we required (i) data availability in all six outlets, (ii) the four selected items to have a total market share of at least 20% in their product category and (iii) the four selected items to show sufficient price variation. Over the whole time span the four items together should show at least 20 price changes of at least 5%, where we counted the typical V-pattern of a price markdown as 1 price change. At least 3 of these price changes should be regular price changes. The minimum market share requirement should make certain that the chosen four items are important within their category. This should raise the relevance of our estimates. Sufficient price variation is an obvious requirement if one wants to estimate a demand curve accurately.
Appendix 3: Identification of Markdowns

Figures 6 and 7 illustrate the identification of markdowns for an individual item of potatoes and lemonade. We define a markdown as a sequence of three, two or one price(s) that are/is below both the most left adjacent price and the most right adjacent price. To calculate our “excluding markdowns” statistics in Section 2, we have filtered out markdown prices. We have replaced them by the last observed regular price.

Figure 6: Price for Potato Item Including and Excluding Temporary Markdowns and Quantities

Figure 7: Price for Lemonade Item Including and Excluding Temporary Markdowns and Quantities
## Appendix 4: Robustness (Fisher price index)

### Table 8: Importance of Demand and Supply Shocks

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Including markdowns</th>
<th>Excl. markdowns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25%</td>
<td>50%</td>
</tr>
<tr>
<td>Average absolute $\Delta \ln(p_i/P^*)$</td>
<td>2%</td>
<td>3%</td>
</tr>
<tr>
<td>Average absolute $\Delta \ln(q_i/Q)$</td>
<td>38%</td>
<td>57%</td>
</tr>
<tr>
<td>Standard Deviation $\Delta \ln(p_i/P^*)$</td>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>Standard Deviation $\Delta \ln(q_i/Q)$</td>
<td>50%</td>
<td>75%</td>
</tr>
<tr>
<td>Correlation ($\Delta \ln(p_i/P^*)$; $\Delta \ln(q_i/Q)$)</td>
<td>-0.45</td>
<td>-0.22</td>
</tr>
<tr>
<td>% Supply Shocks to $\Delta \ln(p_i/P^*)$ (a)</td>
<td>48%</td>
<td>71%</td>
</tr>
<tr>
<td>% Supply Shocks to $\Delta \ln(q_i/Q)$ (a)</td>
<td>50%</td>
<td>70%</td>
</tr>
</tbody>
</table>

Note: The statistics reported in this table are based on bi-weekly data for 2274 items belonging to 58 product categories in six outlets. Individual nominal items prices ($p_i$) are common across the outlets, all the other data ($P^*$, $q_i$, $Q$) can be different per outlet. (a) The contribution of demand shocks to price and quantity variability equals 1 minus the contribution of supply shocks. Computation methods are described in the main text.

### Table 9: Asymmetric Price Sensitivity: Difference between $\varepsilon^H$ and $\varepsilon^L$

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Including markdowns</th>
<th>Excluding markdowns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25%</td>
<td>50%</td>
</tr>
<tr>
<td>Median $\varepsilon^H - \varepsilon^L$</td>
<td>-20.14</td>
<td>-1.27</td>
</tr>
</tbody>
</table>

Note: $\varepsilon^H$ and $\varepsilon^L$ are the absolute values of the price elasticity of demand at high and low relative prices respectively. $\varepsilon^H > \varepsilon^L$ suggests that the demand curve is concave (smoothed "kinked"). The reported data refer to the items at the 25th, 50th and 75th percentile, ordered from low to high.
Appendix 5: Derivation of Curvature in the Behavioral AIDS Model

Starting from Equation (8)

\[ \varepsilon_{i{(LA/B\text{-AIDS})}} = 1 - \frac{\gamma_{ii}}{s_i} + \beta_i - \frac{2\delta_{ii} \ln\left(\frac{p_i}{p}\right)}{s_i} + 2 \sum_{j=1}^{N} \delta_{ij} \ln\left(\frac{p_j}{p}\right) \]

the derivation of the curvature goes as follows:

\[
\begin{align*}
\epsilon_{i{(LA/B\text{-AIDS})}} &= \frac{\partial \ln \varepsilon_i}{\partial \ln p_i} \\
&= -\frac{1}{\varepsilon_i} \frac{\partial}{\partial p_i} \left( \frac{\gamma_{ii} + 2\delta_{ii} \ln\left(\frac{p_i}{p}\right)}{s_i} - 2 \sum_{j=1}^{N} \delta_{ij} \ln\left(\frac{p_j}{p}\right) \right) \\
&= -\frac{1}{\varepsilon_i} \left( 2\delta_{ii}(1-s_i) - \frac{(\partial s_i/\partial \ln p_i)(\gamma_{ii} + 2\delta_{ii} \ln\left(\frac{p_i}{p}\right))}{s_i^2} - 2(\delta_{ii} - s_i \sum_{j=1}^{N} \delta_{ij}) \right) \\
&= -\frac{1}{\varepsilon_i} \left( 2\delta_{ii}(1-s_i) - \left(\varepsilon_i - 1\right) \left( 1 - \varepsilon_i + \beta_i + 2 \sum_{j=1}^{N} \delta_{ij} \ln\left(\frac{p_j}{p}\right) \right) - 2(\delta_{ii} - s_i \sum_{j=1}^{N} \delta_{ij}) \right)
\end{align*}
\]

In the third line we again use the (empirically supported) assumption that \( \frac{\partial \ln p_i}{\partial \ln p_i} = s_i \). The fourth line relies on the definition that \( -\frac{\partial s_i/\partial \ln p_i}{\partial \ln p_i} = (\varepsilon_i - 1) \) and the result derived from Equation (8) that \( \frac{\gamma_{ii} + 2\delta_{ii} \ln\left(\frac{p_i}{p}\right)}{s_i} = 1 - \varepsilon_i + \beta_i + 2 \sum_{j=1}^{N} \delta_{ij} \ln\left(\frac{p_j}{p}\right) \). Rearranging and imposing the steady state assumption that all relative prices are 1, we find for the curvature that

\[
\begin{align*}
\epsilon_{i{(LA/B\text{-AIDS})}} &= \frac{1}{\varepsilon_i} \left( (\varepsilon_i - 1)(\varepsilon_i - 1 - \beta_i) - \frac{2\delta_{ii}(1-s_i)}{s_i} + 2(\delta_{ii} - s_i \sum_{j=1}^{N} \delta_{ij}) \right)
\end{align*}
\]
Appendix 6: Estimation Results for $\delta_{ii}$

The two figures below show the distribution of the 112 (=28x4) estimated values for $\delta_{ii}$ and the distribution of the related absolute t-values. The table contains the results of a Wald test for each of the 28 product categories of the joint hypothesis that $\delta_{11} = \delta_{22} = \delta_{33} = \delta_{44} = 0$. The results are briefly discussed in the main text.

Figure 8: Point Estimates $\delta_{ii}$

Figure 9: t-values $\delta_{ii}$

Table 10: Wald Test for $\delta_{11} = \delta_{22} = \delta_{33} = \delta_{44} = 0$

<table>
<thead>
<tr>
<th>p-value</th>
<th>$p \leq 0.05$</th>
<th>$0.05 &lt; p \leq 0.1$</th>
<th>$0.1 &lt; p \leq 0.2$</th>
<th>$0.2 &lt; p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of product categories</td>
<td>19</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>
Appendix 7: Size of "Other" and Estimation Results

This appendix reveals that there is no specific relationship between our estimation results for the elasticity and the curvature in a product category and the number of items not included in the regressions. The table below contains all relevant correlation coefficients, the figures illustrate two of the results involving curvature.

Table 11: Pairwise Correlation Coefficients over 28 Product Categories

<table>
<thead>
<tr>
<th></th>
<th>Market Share &quot;Other&quot;</th>
<th>Number of Items</th>
<th>Median Elasticity</th>
<th>Median Curvature</th>
<th>Median $\delta_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Share &quot;Other&quot;</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Number of Items</td>
<td>0.61</td>
<td>1</td>
<td>-0.19</td>
<td>0.06</td>
<td>-0.19</td>
</tr>
<tr>
<td>Median Elasticity</td>
<td>-0.06</td>
<td>-0.19</td>
<td>1</td>
<td>0.56</td>
<td>1</td>
</tr>
<tr>
<td>Median Curvature</td>
<td>-0.03</td>
<td>0.06</td>
<td>0.56</td>
<td>1</td>
<td>-0.78</td>
</tr>
<tr>
<td>Median $\delta_{ij}$</td>
<td>0.12</td>
<td>-0.19</td>
<td>0.12</td>
<td>-0.29</td>
<td>-0.78</td>
</tr>
</tbody>
</table>

Figure 10: Curvature vs. Market Share "Other"

Figure 11: Curvature vs. # Items in Product Category