Disentangling the Sources of Inflation Persistence*

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Abstract

We define the sources of inflation persistence at the business cycle frequency as either intrinsic, extrinsic, or expectations-based. To disentangle these different sources of persistence we formulate an unobserved component model and estimate it using the Kalman filter and Bayesian estimation techniques. We find that inflation persistence, expressed as the half-life of a shock, can range from less than one quarter for intrinsic persistence, to several years for extrinsic or expectations-based persistence. This indicates that a significant part of the observed univariate inflation persistence is inherited and that price-setting frictions such as indexation to past inflation or backward-looking expectations are not the best way to match the empirical evidence. We also find evidence that the recent decline in inflation persistence may have originated in a better anchoring of long-run inflation expectations.

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1 Introduction

To design business cycle models that can explain cyclical inflation dynamics we need to know how persistent inflation is. A common practice is to estimate a univariate autoregressive time series model and measure persistence as the sum of the autoregressive coefficients (e.g. Nelson and Plosser, 1982; Fuhrer and Moore, 1995; Pivetta and Reis, 2007). In most of these studies, inflation is found to exhibit high to very high persistence during the post-WW II period, i.e. persistence is found to be close to that of a random walk. More recently, a second empirical observation on inflation persistence emerged. Beechey and Österholm (2007) and Cogley and Sargent (2007) document that inflation persistence rose during the seventies and fell thereafter. Matching this time-variability in inflation persistence forms an additional challenge to the models we design.

We can design models that generate high inflation persistence using a variety of frictions: backward-looking agents (Galí and Gertler, 1999), price indexation (Christiano et al., 2005), consumption habit persistence (Christiano et al, 2005), learning (Milani, 2007), real wage rigidities (Blanchard and Galí, 2007). For a particular calibration these frictions will all be able to replicate the univariate reduced form inflation persistence. But it is important to note that this estimated high persistence is a measure of unconditional inflation persistence. The different frictions that can match the observed inflation persistence will affect the persistence of inflation differently. Some frictions will make inflation directly or intrinsically more persistent, whereas other frictions will make inflation persistent through making its drivers persistent. The drivers of inflation can be read off from the Phillips curve relation that generally emerges in New Keynesian models, and that links current inflation to expectations about future inflation and a measure of the output gap. Frictions that make inflation intrinsically more persistent typically augment the standard Phillips curve relation with lags of inflation.

This paper contributes to the first piece of empirical evidence. We provide evidence that can help identify which frictions are important to match the observed univariate inflation persistence. We disentangle inflation persistence in a number of economically meaningful sources. We loosely follow Angeloni et al. (2006) in distinguishing three sources of cyclical inflation persistence. First, due to price indexation or backward-looking agents inflation can directly be related to its own lags. We call this kind of persistence *intrinsic* inflation persistence. Second, due to asymmetric information (Andolfatto et al., 2007) or imperfect credibility (Erceg and Levin, 2003), private agents' perceptions about the central bank's inflation target can differ from the true inflation target. We call the persistence of such deviations *expectations-based* persistence. Third, inflation persistence is determined by the persistent movements in the output gap. We call this type of persistence *extrinsic* inflation persistence. Both expectations-based and extrinsic persistence can also be labeled inherited inflation persistence, because inflation inherits its persistence from the persistent movements in its driving variables. Each of these three types of inflation persistence represent persistence at the business cycle frequency. On top of this, changes in the long-run inflation rate will add a fourth source of inflation persistence that is not related to the business cycle. For designing business cycle models we are only interested in inflation persistence excluding the effect of changes in the long-run inflation rate.[1] It is widely accepted that this long-run inflation rate is determined by the central bank's inflation target.[2]

We use an unobserved component model to filter out these four different sorts of inflation persistence. This type of model can decompose a time series in a number of distinct components. This is particularly useful given the different sources of inflation persistence we want to measure. See Canova (2007) for how this filtering method compares to other methods for extracting cyclical information from the data. We decompose inflation in a number of components using a univariate and a multivariate identification. We estimate the parameters of the filter using Bayesian estimation techniques. Especially in the multivariate case the number of parameters becomes large so that we get the same problem of overfitting as in the case of VARs (Canova, 2007). To overcome this problem we use prior information from a number of older studies using similar models as ours. We estimate inflation persistence for the euro area and the U.S. The results show that intrinsic inflation persistence is fairly low, i.e. the half-life of a cost-push shock is less than one quarter. We conclude that a significant part of the observed univariate inflation persistence is inherited and that price-setting frictions such as indexation to past inflation or backward-looking expectations are not the best way to match the empirical evidence. We also briefly assess the time-variability of inflation persistence through the lens of our model, and find evidence that the recent decline in cyclical inflation persistence may have originated in a better anchoring of long-run inflation expectations.

2 An Unobserved Component Model

In this section, we present an unobserved component model for inflation which takes into account (i) possible shifts in the central bank's inflation target, (ii) expectations-based persistence, (iii) intrinsic persistence and (iv) extrinsic persistence. The model is identified both in a univariate

[1] Recently, a number of authors has estimated measures of inflation persistence corrected for changes in the long-run inflation rate. See e.g. Altissimo et al. (2006), O'Reilly and Whelan (2005), Pivetta and Reis (2007), Levin and Piger (2004), Ireland (2007), Gadzinski and Orlandi (2004), Cogley and Sargent (2001, 2005), and Benati (2004).

[2] Although inflation targeting is a monetary policy strategy that only emerged in the 1990s, we will still use this framework for the 1970s and 1980s. It enables us to identify the implicit inflation target of central banks from their policy choices.

and a multivariate set-up. The univariate approach only uses information in the time series of inflation. In the multivariate model, we add information contained in real output and the central bank's key interest rate. We use a variant of the Rudebusch and Svensson (1999) model to impose more economic structure. The state space representation of both models is given in Appendix A.

2.1 Baseline Model

The baseline model is given by:

$$\begin{aligned}
\varphi(L)\,\pi_t &= (1-\varphi)\,\pi_t^P + \beta_1 z_{t-1} + \varepsilon_{1t}, &(1)\\
\pi_{t+1}^T &= \pi_t^T + \eta_{1t}, &(2)\\
\pi_{t+1}^P &= E_{t+1}\pi_{t+1}^T, &(3)\\
\beta(L)\,z_t &= \eta_{3t}, &(4)
\end{aligned}$$

where

$$\begin{aligned}
\varphi(L) &= \left(1-\varphi_1 L-\varphi_2 L^2-\varphi_3 L^3-\varphi_4 L^4\right),\\
\beta(L) &= 1-\beta_2 L-\beta_3 L^2,\\
\varphi &= \textstyle\sum_{i=1}^{q}\varphi_i < 1.
\end{aligned}$$

where π_t is the observed inflation rate, π_t^P is the perceived inflation target, π_t^T is the central bank's inflation target and z_t is the output gap, i.e. the percentage deviation of real output from potential output. L is the lag operator so that $L^i\pi_t = \pi_{t-i}$. ε_{1t}, η_{1t} and η_{3t} are mutually independent zero mean white noise processes.

Equation (2) specifies π_t^T as a random walk process, i.e. shifts in the central bank's inflation target are assumed to be permanent. This assumption is consistent with general equilibrium models where long-run inflation is usually pinned down by the inflation target of the central bank.

Shifts in π_t^T are unlikely to be passed on to inflation expectations immediately. Castelnuovo et al. (2003) present data on long-run inflation expectations. These suggest that in the aftermath of shifts in monetary policy, convergence towards the new equilibrium evolves smoothly over time. Even if the central bank clearly announces a new inflation target, it can take quite some time before the new policy target is incorporated into long-run inflation expectations of private agents (Castelnuovo et al., 2003). This is often attributed to asymmetric information and signal extraction or imperfect credibility (see e.g. Erceg and Levin, 2003 and Andolfatto et al., 2007). Agents must then form expectations about the inflation target π_t^T. Equation (3) introduces the perceived inflation target π_t^P, which captures the private agents' beliefs about the central bank's inflation target π_t^T.

The expectations operator in equation (3) is parameterized by defining π_{t+1}^P as a weighted average of $\pi_t^P$and π_{t+1}^T,

$$
\begin{aligned}
\delta\left(L\right)\pi_{t+1}^P &= \delta\pi_{t+1}^T+\eta_{2t}, \qquad (5)\\
\text{where } \delta\left(L\right) &= \left(1-\left(1-\delta\right)L\right), \quad 0<\delta\leq 1,
\end{aligned}
$$

where η_{2t} is a zero mean white noise process. Note that shocks to the perceived inflation target, η_2, only have a short-run impact on π^P. These shocks should be interpreted as misperceptions of private agents about the central bank's inflation target. Shocks to the central bank's inflation target, η_1, have a unit long-run impact on π^P, i.e. π^T is the long-run inflation rate. This is consistent with the widely accepted view that long-run inflation is a purely monetary phenomenon.

Equation (1) is a Phillips curve relation, linking the observed inflation rate π_t to the perceived inflation target π_t^P, q lags of inflation and the lagged output gap z_{t-1}. The perceived inflation target π_t^P is the inflation rate consistent with the private agents' inflation expectations. Therefore, it serves as the medium-run inflation anchor. Both shocks to the output gap z_{t-1} and cost-push shocks ε_{1t} induce temporary deviations of π_t from π_t^P. The sluggish adjustment of π_t in response to cost-push shocks ε_{1t} is measured by the sum of the AR coefficients, φ. The sluggish adjustment of π_t in response to output gap shocks is determined, besides the intrinsic inflation persistence, by the persistence of the output gap z_t. We call this source of inflation persistence extrinsic inflation persistence.

2.2 Univariate Model

In a first step, we only use inflation data to estimate the unobserved component model in (1)-(5). As inflation is composed of different components driven by 4 unobserved disturbance terms $(\varepsilon_{1t},\eta_{1t},\eta_{2t},\eta_{3t})$, it is not immediately obvious that the model is identified from observed inflation only. In order to show that this model is identified, consider the single equation or UCARIMA form[3]

$$\varphi\left(L\right)\pi_t=\left(1-\varphi\right)\delta\left(L\right)^{-1}L\left(\delta\Delta^{-1}\eta_{1t}+\eta_{2t}\right)+\beta_1\beta\left(L\right)^{-1}L\eta_{3t}+\varepsilon_{1t},$$

[3]UCARIMA stands for Unobserved Component Auto-Regressive Integrated Moving Average process.

where we use $\pi_t^P = \delta(L)^{-1} L \left(\delta\Delta^{-1}\eta_{1t} + \eta_{2t}\right)$ and $z_t = \beta(L)^{-1}\eta_{3t}$. The model can be made stationary by multiplying through by $\delta(L)\beta(L)\Delta$

$$\begin{aligned}
\alpha(L)\Delta\pi_t &= (1-\varphi)\beta(L)L(\delta\eta_{1t} + \Delta\eta_{2t}) + \beta_1\delta(L)L\Delta\eta_{3t} + \delta(L)\beta(L)\Delta\varepsilon_{1t}, \qquad (6)\\
\text{where } \alpha(L) &= \delta(L)\beta(L)\varphi(L).
\end{aligned}$$

Given that the model is linear the components driven by the disturbances $\varepsilon_{1t}, \eta_{1t}, \eta_{2t}$ and η_{3t} can be combined to give a model with a single disturbance e_t, known as the *reduced form* or *canonical form* (see Harvey, 1985). The reduced form of (6) is a restricted ARIMA model

$$\alpha(L)\Delta\pi_t = \theta(L)e_t, \qquad (7)$$

As the autoregressive polynomials $\delta(L), \beta(L)$ and $\varphi(L)$ contain no common factors, the order of the polynomials $\alpha(L)$ and $\theta(L)$ can be determined by comparing equations (7) and (6). The order of the autoregressive polynomial $\alpha(L)$ is given by the sum of the orders of $\delta(L), \beta(L)$ and $\varphi(L)$, which is 7. The order of the moving average polynomial $\theta(L)$ is given by the maximum of the orders of the 4 moving average components on the right-hand side of (6), which is 4. The reduced form in (7) is therefore an ARIMA$(7,1,4)$ model. This reduced form has 12 parameters, which is sufficient to identify the 12 parameters in the model in (1)-(5).

2.3 Multivariate Model

Although the univariate model is identified, using information in other time series should help the measurement of inflation persistence and at the same time serve as a robustness check. In the univariate model identification of shocks to the central bank's inflation target and to the ouput gap both stem from the purely statistical restrictions that the former shocks should have a unit long-run impact on inflation while the latter should have a cyclical impact. We try to improve the identification of the components driving inflation combining economic restrictions and additional data on the central bank's key interest rate and real output. We use a variant of the model in Rudebusch and Svensson (1999) to (i) identify the central bank's inflation target from information contained in the central bank's key interest rate and (ii) to measure extrinsic inflation persistence in response to shocks to the output gap from information contained in real output. The specification in equations (2)-(5) now becomes:

$$\begin{aligned}
\varphi(L)\pi_t &= (1-\varphi)\pi_t^P + \beta_1 z_{t-1} + \varepsilon_{1t}, && (8)\\
\pi_{t+1}^T &= \pi_t^T + \eta_{1t} && (9)\\
\delta(L)\pi_{t+1}^P &= \delta\pi_{t+1}^T + \eta_{2t} && (10)\\
i_t &= \rho_2 i_{t-1} + (1-\rho_2)\left(r_t^* + \pi_t^P\right) + \rho_1(\pi_{t-1} - \pi_t^T) + \varepsilon_{2t} && (11)\\
y_t^r &= y_t^P + z_t && (12)\\
z_t &= \beta_2 z_{t-1} + \beta_3 z_{t-2} - \beta_4\left(i_{t-1} - \pi_{t-1}^P - r_{t-1}^*\right) + \varepsilon_{3t} && (13)\\
y_{t+1}^P &= \lambda_{t+1} + y_t^P + \eta_{3t}' && (14)\\
\lambda_{t+1} &= \lambda_t + \eta_{4t} && (15)\\
r_{t+1}^* &= \gamma\lambda_{t+1} + \tau_{t+1} && (16)\\
\tau_{t+1} &= \theta\tau_t + \eta_{5t} && (17)
\end{aligned}$$

where

$$\begin{aligned}
\varphi(L) &= \left(1 - \varphi_1 L - \varphi_2 L^2 - \varphi_3 L^3 - \varphi_4 L^4\right) && (18)\\
\beta(L) &= 1 - \beta_2 L - \beta_3 L^2, && (19)\\
\varphi &= \sum\nolimits_{i=1}^{q} \varphi_i < 1 && (20)\\
\delta(L) &= (1 - (1-\delta)L), \quad 0 < \delta \leq 1 && (21)
\end{aligned}$$

We assume that all shocks are mutually independent zero mean white noise processes. Now that output is an observable variable we replace η_{3t} by ε_{3t}, and we label the shock to potential output as η_{3t}'. The interest rate rule in equation (11) infers on the stance of monetary policy through comparing the central bank's key nominal interest rate, i_t, with a measure for the neutral stance of monetary policy. Following Laubach and Williams (2003), this measure is assumed to be the natural short-run nominal interest rate $\left(r_t^* + \pi_t^P\right)$, where r_t^* is the time-varying real short-term interest rate consistent with output equal to potential. As the perceived inflation target π_t^P is the medium-run inflation anchor consistent with long-run inflation expectations, $r_t^* + \pi_t^P$ is the medium-run nominal interest rate anchor for monetary policy. The term $(\pi_{t-1} - \pi_t^T)$ captures the reaction of the central bank to deviations of inflation from its target, i.e. monetary authorities will increase the nominal interest rate i_t when observed inflation π_{t-1} lies above the inflation target π_t^T. The lagged interest rate i_{t-1} introduces a degree of nominal interest rate smoothing or policy inertia (see e.g. Amato and Laubach, 1999; English et al., 2003; Erceg and Levin, 2003). We assume that the policy parameters ρ_1 and ρ_2 are time-invariant. Although Clarida et al. (1998) find that the policy parameters are unstable in a number of countries, this assumption is not in contradiction with their results. They estimate the parameters conditional on a constant inflation target, whereas we estimate the inflation target conditional on constant policy parameters. Both strategies are observationally equivalent. The reason why we do so is that we are interested in the implied time-varying inflation target and not in the policy parameters. For examples of the same approach see e.g. Kozicki and Tinsley (2005) or Smets and Wouters (2005).

Equation (12) decomposes the log of real output y_t^r into potential output y_t^P and the output gap z_t. Equation (13) relates the output gap z_t to its own lags and a term $\left(i_{t-1} - \pi_{t-1}^P - r_{t-1}^*\right)$ which captures monetary policy transmission. Following Harvey (1985), Stock and Watson (1998) and Laubach and Williams (2003), equations (14)-(15) model potential output as a random walk with drift, where the drift term λ_t varies over time according to a random walk process. The time-variation in λ_t allows for the possibility of permanent changes in the trend growth of real output, e.g. the productivity slowdown of the early 1970s.

Laubach and Williams (2003) argue that the natural real rate of interest varies over time due to shifts in the trend growth of output and other factors such as households' rate of time preference. Therefore, equation (16) relates the real short-term interest rate r_t^* to the trend growth in potential output λ_t and a component τ_t that captures other determinants like time preferences. τ_t is assumed to be an AR process that, depending on the value for θ, can be either stationary or non-stationary.

3 Data, Estimation and Results

3.1 Data

We use quarterly data for the euro area and the United States from 1970:1 to 2005:4.[4] The inflation series π_t is the annualized first difference of the log of the seasonally adjusted GDP deflator. For the interest rate, i_t, we use the annualized central bank key interest rate. This interest rate should be most appropriate to infer changes in the central bank's behavior. Real output, y_t^r, is measured as the log of seasonally adjusted GDP at constant prices. See Appendix B for a more detailed data description. We set the number of AR terms in equation (1) equal to 4, i.e. $q = 4$.

[4] Although the euro area did not exist for the larger part of our data sample (1970:2-1998:4), we use synthetic data aggregating the national data (Fagan et al., 2005). We implicitly assume that the euro area was an economy with a homogeneous monetary policy over the entire sample.

3.2 Bayesian Estimation

To calculate the likelihood function of our model we write it in state space form (see the Appendix) and use the Kalman filter. The Kalman filter algorithm requires that parameter vector ψ of the model is known. One approach is to derive, from the exact Kalman filter, the diffuse loglikelihood function for our model (de Jong, 1991; Koopman and Durbin, 2000; Durbin and Koopman, 2001) and replace the unknown parameter vector ψ by its maximum likelihood estimate. This is not the approach we pursue here. First, given the fairly large number of unknown parameters, especially in the multivariate model, the numerical optimization of the sample loglikelihood function is difficult. As a lot of the unknown parameters in ψ were estimated previously for different countries and samples we analyze the state space models from a Bayesian point of view, i.e. we treat ψ as a random parameter vector with a known prior density $p(\psi)$. We estimate the posterior densities $p(\psi \mid y, x)$ for the parameter vector ψ, by combining prior information contained in $p(\psi)$ and sample data in y and x denoting vectors of data for the endogenous and exogenous variables. This boils down to calculating the posterior mean $\overline{g}$:

$$\overline{g} = E\left[g(\psi) \mid y, x\right] = \int g(\psi)\, p(\psi \mid y, x)\, d\psi \tag{22}$$

where g is a function which expresses the moments of the posterior densities $p(\psi \mid y, x)$ in terms of the parameter vector ψ. See the Appendix for more details on the estimation method.

Prior Information

We include prior information about the unknown parameter vector ψ in Table 1 through the prior density $p(\psi)$. Where possible prior information is taken from previous studies. We use the same priors for the euro area and the United States. If no adequate information is available, we leave considerable uncertainty around the chosen priors. The prior distribution is assumed to be Gaussian for all elements in ψ, except for the variance parameters, which are assumed to be gamma distributed.

The priors for the AR coefficients φ_i are chosen from studies allowing for a break in the mean of the inflation rate. Levin and Piger (2004) for instance find a value of 0.36 for the sum of the AR coefficients of the United States GDP deflator. Gadzinski and Orlandi (2004) find a somewhat higher figure of 0.6 for the euro area. We choose a prior for the sum of the AR coefficients of 0.4 for both the United States and the euro area. As these parameter values are important for our question, we leave a large degree of uncertainty around the priors values. Our prior for δ is 0.15, which is close to the parameter values determining signal extraction in Erceg and Levin

(2003) and Kozicki and Tinsley (2005). The prior for the variance of the inflation target shocks $\sigma^2_{\eta_1}$ is close to the evidence Kozicki and Tinsley (2005) and Smets and Wouters (2005) find. The priors for the parameters that are only present in the multivariate model come from previous studies estimating variants of the model of Rudebusch and Svensson (1999). For the impact of the lagged output gap on inflation we choose a value of 0.2. As we use annualized quarterly inflation this value is consistent with a lot of studies reporting a value of 0.05 for the output gap impact on the inflation rate. The AR coefficients of the output gap equation are chosen in order to generate a hump-shaped response of output in reaction to a shock. This feature is often found in previous empirical studies (Gerlach and Smets, 1999; Rudebusch and Svensson, 1999; Laubach and Williams, 2003; Rudebusch, 2005). The parameter value for ρ_2 assumes considerable interest rate smoothing (Smets and Wouters, 2005). The parameter values for ρ_1 and ρ_2 are chosen so that the Taylor (1993) principle $\left(1 + \frac{\rho_1}{1-\rho_2} = 1.5 > 1\right)$ holds for deviations of π_t^P from π_t^T. The central bank reacts less vigorously $\left(\frac{\rho_1}{1-\rho_2} = 0.5\right)$ in response to deviations of π_t from π_t^T. This is consistent with the view that an inflation-targeting central bank should stabilize inflation in the medium run and pay less attention to short-term deviations.

Table 1: Prior information

	Reference(s)	5%	Mean	95%
φ_1	-	0.04	0.20	0.36
φ_2	-	-0.06	0.10	0.26
φ_3	-	-0.11	0.05	0.21
φ_4	-	-0.11	0.05	0.21
$\sum_{i=1}^{4} \varphi_1$	Gadzinski et al. (2004), Levin et al. (2004)	0.16	0.40	0.64
δ	Erceg et al. (2003), Kozicki et al. (2003)	-0.01	0.15	0.31
β_1	Gerlach et al. (1999), Rudebusch (2005), Rudebusch et al. (1999)	0.18	0.20	0.22
β_2	Gerlach et al. (1999), Rudebusch (2005), Rudebusch et al. (1999)	1.32	1.35	1.38
β_3	Gerlach et al. (1999), Rudebusch (2005), Rudebusch et al. (1999)	-0.50	-0.47	-0.44
β_4	Gerlach et al. (1999), Rudebusch (2005), Rudebusch et al. (1999)	-0.01	0.15	0.31
ρ_1	Taylor (1993)	0.02	0.05	0.08
ρ_2	Taylor (1993), Smets et al. (2005)	0.87	0.90	0.93
γ	Laubach et al. (2003)	3.67	4.00	4.33
θ	Laubach et al. (2003)	0.95	0.97	0.99
$\sigma^2_{\varepsilon_1}$	-	0.35	1.30	2.77
$\sigma^2_{\varepsilon_2}$	-	0.21	0.30	0.40
$\sigma^2_{\varepsilon_3}$	Laubach et al. (2003)	0.11	0.16	0.21
$\sigma^2_{\eta_1}$	Kozicki et al. (2003), Smets et al. (2005)	0.03	0.12	0.25
$\sigma^2_{\eta_2}$	-	2.8e-5	1.0e-4	2.1e-4
$\sigma^2_{\eta_3}$	-	0.4e-3	1.5e-3	3.4e-3
$\sigma^{2\prime}_{\eta_3}$	Laubach et al. (2003)	0.26	0.37	0.49
$\sigma^2_{\eta_4}$	Laubach et al. (2003)	4.5e-4	6.5e-4	8.8e-4
$\sigma^2_{\eta_5}$	Laubach et al. (2003)	0.07	0.10	0.14

Note: All variances are expressed at annual rates except for $\sigma^2_{\varepsilon_3}$, $\sigma^2_{\eta_3}$ and $\sigma^2_{\eta_4}$ which are expressed as quarterly rates. The prior distribution is assumed to be Gaussian for all elements in ψ, except for the variance parameters which are assumed to be gamma distributed.

3.3 Results

3.3.1 Posterior Distribution of the Parameters

Tables 2 and 3 present the posterior mean and the 5th and 95th percentile of the posterior distribution of ψ for the euro area and the United States for both the univariate and multivariate model. Two important conclusions stand out. First, in the univariate model intrinsic inflation persistence, measured as $\sum_{i=1}^{q} \varphi_i$, is 0.42 for the euro area and 0.80 for the United States. This is considerably lower than estimates from standard AR time series models. The multivariate intrinsic inflation persistence estimates are 0.45 and 0.73 for the euro area and the United States. This is in line with the results of the univariate specification. In the case of the United States, intrinsic inflation persistence is somewhat higher than in the euro area. Note that this result is consistent with Galí et al. (2001), who for the United States also find a relatively higher degree of backward-lookingness compared to the euro area. Second, expectations-based persistence, measured by $(1-\delta)$, is at least as high or higher than intrinsic inflation persistence, i.e. higher than 0.73 for both economies across the different models. The persistence in the output gap, measured by the sum of β_2 and β_3, is close to 0.9. This implies considerable extrinsic inflation persistence.

Table 2: Posterior Distribution Univariate Model

	Euro area			United States		
	5%	Mean	95%	5%	Mean	95%
φ_1	0.14	0.25	0.36	0.24	0.37	0.49
φ_2	0.00	0.11	0.22	0.06	0.17	0.28
φ_3	-0.18	-0.07	0.03	0.03	0.14	0.25
φ_4	0.03	0.13	0.23	0.02	0.12	0.23
$\sum_{i=1}^{4} \varphi_i$	**0.20**	**0.42**	**0.63**	**0.59**	**0.80**	**0.98**
δ	0.15	0.23	0.35	0.00	0.16	0.33
β_2	1.19	1.33	1.47	1.21	1.35	1.49
β_3	-0.64	-0.49	-0.35	-0.63	-0.49	-0.35
$\sigma^2_{\varepsilon_1}$	1.29	1.58	1.96	1.10	1.37	1.69
$\sigma^2_{\eta_1}$	0.09	0.16	0.29	0.02	0.08	0.22
$\sigma^2_{\eta_2}$	5.0e-5	1.0e-4	2.0e-4	5.0e-5	1.0e-4	2.0e-4
$\sigma^2_{\eta_3}$	0.01	0.02	0.07	0.01	0.03	0.09

Note: The approximate covariance matrix $\widehat{\Omega}$ is inflated with a factor 1.5. For the US, the coefficient of variation of the weights stabilized after 3 updates of the importance function. For the Euro area updating was not necessary. With $n = 10000$, the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ is well below 10% for all coefficients.

Table 3: Posterior Distribution Multivariate Model

	Euro area			United States		
	5%	Mean	95%	5%	Mean	95%
φ_1	0.15	0.25	0.35	0.19	0.30	0.40
φ_2	0.01	0.11	0.21	0.04	0.14	0.24
φ_3	-0.17	-0.07	0.03	0.04	0.14	0.25
φ_4	0.06	0.16	0.26	0.05	0.15	0.25
$\sum_{i=1}^{4} \varphi_i$	**0.25**	**0.45**	**0.64**	**0.59**	**0.73**	**0.88**
δ	0.16	0.27	0.40	0.11	0.27	0.43
β_1	0.16	0.22	0.29	0.15	0.21	0.28
β_2	1.23	1.34	1.44	1.26	1.36	1.47
β_3	-0.48	-0.37	-0.27	-0.53	-0.43	-0.32
β_4	0.01	0.02	0.02	0.01	0.01	0.02
ρ_1	0.01	0.04	0.07	0.01	0.05	0.09
ρ_2	0.85	0.89	0.92	0.85	0.88	0.92
γ	3.38	3.94	4.49	3.42	3.99	4.55
θ	0.95	0.97	0.99	0.95	0.97	0.99
$\sigma^2_{\varepsilon_1}$	1.23	1.51	1.87	0.99	1.20	1.48
$\sigma^2_{\varepsilon_2}$	0.23	0.28	0.34	0.70	0.81	0.94
$\sigma^2_{\varepsilon_3}$	0.08	0.11	0.14	0.10	0.14	0.19
$\sigma^2_{\eta_1}$	0.03	0.08	0.18	0.03	0.08	0.21
$\sigma^2_{\eta_2}$	1.3e-5	5.2e-5	1.6e-4	1.3e-5	5.2e-5	1.6e-4
$\sigma^{2\prime}_{\eta_3}$	0.14	0.18	0.24	0.25	0.32	0.41
$\sigma^2_{\eta_4}$	4.4e-4	6.2e-4	8.5e-4	4.3e-4	6.2e-4	8.6e-4
$\sigma^2_{\eta_5}$	0.07	0.10	0.14	0.08	0.11	0.15

Note: The approximate covariance matrix $\widehat{\Omega}$ is inflated with a factor 1.5. The coefficient of variation of the weights stabilized after 1 update of the importance function for both the euro area and the United States. With $n = 10000$, the probabilistic error bound for the importance sampling estimator $\overline{g}_n$is well below 10% for all coefficients.

3.3.2 Half-Life and Impulse Response Analysis

An alternative way of analyzing inflation persistence is to look at the half life and impulse response functions of different shocks to inflation. The half-life counts the number of periods for which the effect of a shock to inflation remains above half its initial impact. An important difference with the sum of estimated AR coefficients as a measure of persistence is that both the half life and impulse response analysis take all the roots of the AR equation into account while the sum of AR coefficients only measures the average speed of convergence. A second important difference with the point estimates of the AR coefficients is that different sources of persistence in response to a shock can reinforce each other. The inflation dynamics in response to a shock will thus not only depend on the persistence in the variable that was shocked, but will also depend on the interaction with other variables. Therefore, also the persistence in the latter will play a role.

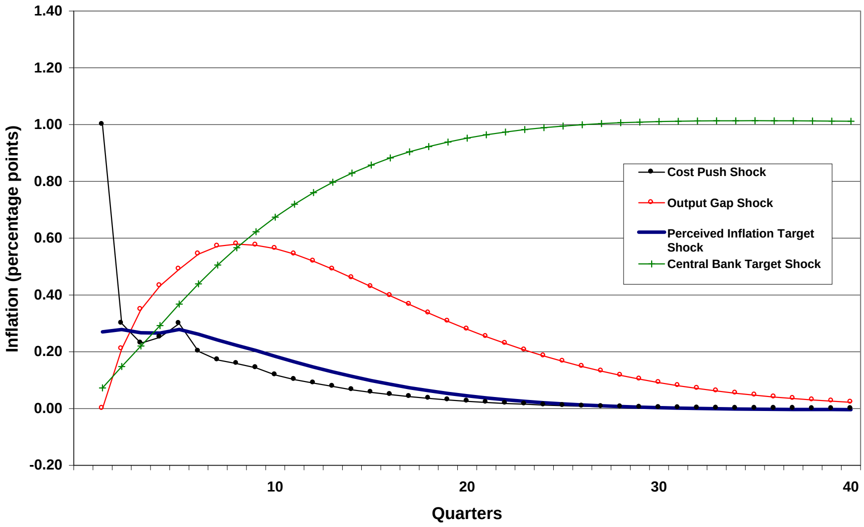
Table 4 reports half lives for four shocks to inflation considered in the multivariate model. The half life of a cost push shock (ε_{1t}) is less than one quarter. For a shock to the perceived inflation target (η_{2t}), the half life is 5 and 11 quarters in the euro area and the United States respectively. For a shock to the output gap (ε_{3t}), the half life even amounts to 35 quarters in the euro area and to 27 quarters the United States. Finally, a shock to the inflation target (η_{1t}) is permanent and therefore its half life is equal to infinity. The latter result is obtained by construction because we assume a random walk process for the shifts in the central bank's inflation target. It shows that ignoring a component with an infinite half life must create a considerable bias in the estimates of the other kinds of persistence.

Table 4: Half Lives of Inflation (Quarters)

	Euro area	United States
Cost Push Shock	0	0
Perceived Inflation Target Shock	5	11
Output Gap Shock	35	27
Central Bank Target Shock	∞	∞

The responses to a unit shock convey the same message in Figure 1. A shift in the central bank's inflation target (η_{1t}) has a permanent impact on inflation. Still, it takes various periods before the inflation rate stabilizes at the new target, both in the euro area and in the United States. This is to a big extent due to considerable expectations-based persistence that creates persistent deviations of the perceived inflation target from the central bank's inflation target. In case of a shock to the output gap (ε_{3t}) or the perceived inflation target (η_{2t}), the response of inflation seems to be characterized by a similar degree of persistence. In case of a temporary

Figure 1: **Impulse Responses United States**



Note: The responses to a unit shock for the euro area are similar to those for the United States. To save space we do not report them here.

shock to inflation (ε_{1t}), the convergence to the target goes much faster. According to the sum of the AR coefficients, intrinsic and expectations-based persistence are not statistically significantly different. Still, due to the persistence in the reaction of the central bank and the output gap, the number of quarters that inflation is affected by a difference between the perceived and the central bank's inflation target can be considerably higher.

3.3.3 Assessing the Recent Decrease in Inflation Persistence

Beechey and Österholm (2007) and Cogley and Sargent (2007) document that during the last two decades there has been a significant drop in inflation persistence. As we do not focus on the time-variability of inflation persistence, we do not allow for stochastic volatility or time-varying coefficients. Nevertheless, if inflation persistence is truly time-varying this should be reflected in the shocks we estimate. By looking at the data with our model we can find an indication of what might be a likely source of the decrease in the inflation persistence. To do this we verify whether the relative size of the estimated smoothed shocks was different before and after 1984:1. We choose this discrete date because a number of authors have found evidence that US and UK time series exhibit a significant drop in volatility after this break date to which they refer as the Great Moderation.[5] We compare the standard deviations of the estimated shocks before and after the break date. If the standard deviations of all shocks drop to the same extent, then there should not be a change in the inflation persistence we measure. However, if in the second period the relative contribution of shocks has changed compared to the first period, then we should

[5] See e.g. Stock and Watson (2002), Benati (2007).

expect a change in univariate inflation persistence. If the standard deviation of shocks giving rise to higher persistence falls more than that of other shocks this explains - in an accounting sense - the recent decrease in inflation persistence.

Table 5: Standard Deviation of Estimated Shocks

Model	Period	Cost-Push	Output Gap	Perceived Target	Central Bank Target
		Euro area			
Univariate	Pre-1984	1.71	0.03	0.02	0.10
	Post-1984	0.84	0.01	0.01	0.07
	Ratio Post/Pre	**0.49**	**0.54**	**0.27**	**0.71**
Multivariate	Pre-1984	1.67	0.07	0.02	0.06
	Post-1984	0.82	0.07	0.00	0.04
	Ratio Post/Pre	**0.50**	**1.00**	**0.16**	**0.73**
		United States			
Univariate	Pre-1984	1.65	0.06	0.07	0.04
	Post-1984	0.83	0.02	0.01	0.02
	Ratio Post/Pre	**0.50**	**0.34**	**0.22**	**0.48**
Multivariate	Pre-1984	1.44	0.11	0.01	0.05
	Post-1984	0.78	0.06	0.00	0.04
	Ratio Post/Pre	**0.54**	**0.52**	**0.66**	**0.70**

Note: The Pre-1984 part of the sample runs from 1971:3 to 1983:4. Five observations are lost due to initialization. The Post-1984 part of the sample runs from 1984:1 to 2005:4. To save space we do not report the remaining shocks. The standard deviation of these shocks also fell by roughly a half. There was no particular pattern in the drop across these shocks.

In Table 5 we see that the standard deviation of all four shocks that we report here has dropped significantly after 1984. This is in line with the evidence on the Great Moderation. A second observation is that the innovation variance of long-run inflation expectations has dropped relatively more than the other sources of inflation persistence. This is true for the evidence from the univariate and multivariate model for the euro area and the univariate model for the United States. We interpret this as evidence of a better anchoring of long-run inflation expectations. As this is a relatively persistent source of inflation persistence, this indicates that inflation persistence can have dropped because of a better anchoring of long-run inflation expectations. This is in line with Cogley and Sargent (2007) who document a decline in cyclical inflation persistence. Beechey and Österholm (2007) attribute the decline in univariate inflation persistence to a change in the preferences of the Federal Reserve, which in our model would show up in a drop in the innovation variance of the central bank's inflation target. As we do not observe a more pronounced drop in the innovation variance of the central bank's inflation target, our evidence suggests that it is rather the cyclical inflation persistence that fell through better anchored long-run inflation expectations. An extension of our model with time-varying coefficients and stochastic volatility seems promising to deliver more evidence on the sources of time-variability in inflation persistence.

4 Conclusion

To design realistic business cycle models that can explain cyclical inflation dynamics we need to know how persistent inflation is. A common practice is to estimate a univariate autoregressive time series model and measure persistence as the sum of the autoregressive coefficients (e.g. Nelson and Plosser, 1982; Fuhrer and Moore, 1995; Pivetta and Reis, 2007). In most of these studies, inflation is found to exhibit high to very high persistence over the post-WW II period, i.e. persistence is found to be close to that of a random walk.

But it is important to note that this estimated high persistence is a measure of unconditional inflation persistence. Our paper extends the set of statistical inflation properties by disentangling inflation persistence in a number of economically meaningful sources. We use an unobserved component model to filter out four different sorts of inflation persistence. We estimate inflation persistence for the euro area and the U.S. The results show that intrinsic inflation persistence is fairly low, i.e. the half-life of a cost-push shock is less than one quarter. We conclude that a significant part of the observed univariate inflation persistence is inherited and that price-setting frictions such as indexation to past inflation or backward-looking expectations are not the best way to match the empirical evidence. We also briefly assess the time-variability of inflation persistence through the lens of our model, and find evidence that the recent decline in cyclical inflation persistence may have originated in a better anchoring of long-run inflation expectations.

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Appendix A: Details on Estimation Method[6]

A.1 State Space Representation

The unobserved component models outlined in section 2 both include a number of unobserved components ($\pi_t^P, \pi_t^T, ...$). In order to estimate these models, it is necessary to write them into state space form[7]. In a state space model, the development over time of the system under study is determined by an unobserved series of vectors $\alpha_1, \ldots, \alpha_n$, which are associated with a series of observed vectors $y_1, \ldots, y_n$. A general linear Gaussian state space model can be written in the following form:

$$y_t = Z\alpha_t + Ax_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, H), \tag{A.1}$$

$$\alpha_{t+1} = T\alpha_t + R\eta_t, \qquad \eta_t \sim N(0, Q), \qquad t = 1, \ldots, n, \tag{A.2}$$

where y_t is a $p \times 1$ vector of observed endogenous variables, modelled in the observation equation $(A.1)$, x_t is a $k \times 1$ vector of observed exogenous variables and α_t is a $m \times 1$ vector of unobserved states, modelled in the state equation $(A.2)$. The disturbances ε_t and η_t are assumed to be independent sequences of independent normal vectors. The matrices Z, A, T, R, H, and Q are parameter matrices.[8]

Univariate Model

$$y_t = \begin{bmatrix} \pi_t \end{bmatrix}; \ \alpha_t = \begin{bmatrix} \pi_t^T & \pi_t^P & \beta_1 z_{t-1} & \beta_1 z_{t-2} \end{bmatrix}'; \ x_t = \begin{bmatrix} \pi_{t-1} & \pi_{t-2} & \pi_{t-3} & \pi_{t-4} \end{bmatrix}';$$

$$Z = \begin{bmatrix} 0 & (1-\varphi) & 1 & 0 \end{bmatrix}; \ A = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \end{bmatrix}; \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \end{bmatrix};$$

$$\eta_t = \begin{bmatrix} \eta_{1t} & \eta_{2t} & \eta_{3t} \end{bmatrix}'; \ H = \begin{bmatrix} \sigma_{\varepsilon_1}^2 \end{bmatrix};$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \delta & (1-\delta) & 0 & 0 \\ 0 & 0 & \beta_2 & \beta_3 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \ R = \begin{bmatrix} 1 & 0 & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; \ Q = \begin{bmatrix} \sigma_{\eta_1}^2 & 0 & 0 \\ 0 & \sigma_{\eta_2}^2 & 0 \\ 0 & 0 & \sigma_{\eta_3}^2 \end{bmatrix}$$

Multivariate Model

$$y_t = \begin{bmatrix} \pi_t & i_t & y_t^r \end{bmatrix}'; \ x_t = \begin{bmatrix} \pi_{t-1} & \pi_{t-2} & \ldots & \pi_{t-q} & y_{t-1} & y_{t-2} & i_{t-1} \end{bmatrix}';$$

$$\alpha_t = \begin{bmatrix} \pi_t^T & \pi_t^P & \pi_{t-1}^P & y_t^P & y_{t-1}^P & y_{t-2}^P & \lambda_t & \lambda_{t-1} & \tau_t & \tau_{t-1} \end{bmatrix}';$$

[6] The method outlined in this section was implemented using a set of GAUSS procedures. The code of these procedures is available from the authors on request.

[7] See e.g. Durbin and Koopman (2001) for an extensive overview of state space methods.

[8] The exact elements of the vectors y_t, x_t and α_t and the matrices Z, A, T, R, H, and Q for both the univariate and the multivariate model are specified in Appendix B.

$$A = \begin{bmatrix} \varphi_1 & \varphi_2 & \dots & \varphi_q & \beta_1 & 0 & 0 \\ \rho_1 & 0 & \dots & 0 & 0 & 0 & \rho_2 \\ 0 & 0 & \dots & 0 & \beta_2 & \beta_3 & -\beta_4 \end{bmatrix};$$

$$Z = \begin{bmatrix} 0 & (1-\sum_{i=1}^{q}\varphi_i) & 0 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ -\rho_1 & (1-\rho_2) & 0 & 0 & 0 & 0 & (1-\rho_2)\gamma & 0 & (1-\rho_2) & 0 \\ 0 & 0 & \beta_4 & 1 & -\beta_2 & -\beta_3 & 0 & \beta_4\gamma & 0 & \beta_4 \end{bmatrix};$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & (1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \quad R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \delta & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} & \varepsilon_{2t} & \varepsilon_{3t} \end{bmatrix}'; \; \eta_t = \begin{bmatrix} \eta_{1t} & \eta_{2t} & \eta_{3t}' & \eta_{4t} & \eta_{5t} \end{bmatrix}';$$

$$H_t = \begin{bmatrix} \sigma^2_{\varepsilon_1} & 0 & 0 \\ 0 & \sigma^2_{\varepsilon_2} & 0 \\ 0 & 0 & \sigma^2_{\varepsilon_3} \end{bmatrix}; \; Q_t = \begin{bmatrix} \sigma^2_{\eta_1} & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{\eta_2} & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\eta_3} & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\eta_4} & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\eta_5} \end{bmatrix}$$

A.2 Kalman Filter and Smoother

Assuming that Z, A, T, R, H, and Q are known, the purpose of state space analysis is to infer the relevant properties of the α_t's from the observations $y_1, \dots, y_n$ and $x_1, \dots, x_n$. This can be done through the subsequent use of two recursions, i.e. the Kalman filter and the Kalman smoother. The objective of filtering is to obtain the distribution of α_t, for $t = 1, \dots, n$, conditional on Y_t and X_t, where $Y_t = \{y_1, \dots, y_t\}$ and $X_t = \{x_1, \dots, x_t\}$. In a linear Gaussian state space model, the distribution of α_t is entirely determined by the filtered state vector $a_t = E(\alpha_t \mid Y_t, X_t)$ and the filtered state variance matrix $P_t = Var(\alpha_t \mid Y_t, X_t)$. The (contemporaneous) Kalman filter algorithm (see e.g. Hamilton, 1994, or Durbin and Koopman, 2001) estimates a_t and P_t by updating, at time t, a_{t-1} and P_{t-1} using the new information contained in y_t and x_t. The Kalman filter recursion can be initialized by the assumption that $\alpha_1 \sim N(a_1, P_1)$. In practice, a_1 and P_1 are generally not known though. Therefore, we assume that the distribution of the initial state vector α_1 is

$$\alpha_1 = V\Gamma + R_0\eta_0, \qquad \eta_0 \sim N(0, Q_0), \qquad \Gamma \sim N(0, \kappa I_r), \tag{A.3}$$

where the $m \times r$ matrix V and the $m \times (m-r)$ matrix R_0 select the r elements of the state vector that are non-stationary and the $m-r$ elements that are stationary respectively. They are composed of columns of the identity matrix I_m and are defined so that, when taken together, their columns constitute all the columns of I_m and $V'R_0 = 0$. The unconditional variance matrix Q_0 of the stationary elements of the state vector is positive definite and can be computed from the model parameters. The $r \times 1$ vector Γ is a vector of unknown random quantities which, as we let $\kappa \to \infty$, is referred to as the diffuse vector. This leads to

$$\alpha_1 \sim N(0, P_1), \qquad P_1 = \kappa P_\infty + P_*, \tag{A.4}$$

where $P_\infty = VV'$ and $P_* = R_0 Q_0 R_0'$. The Kalman filter is modified to account for this diffuse initialization implied by letting $\kappa \to \infty$ by using the exact initial Kalman filter introduced by Ansley and Kohn (1985) and further developed by Koopman (1997) and Koopman and Durbin (2003).

Subsequently, the Kalman smoother algorithm is used to estimate the distribution of α_t, for $t = 1, \ldots, n$, conditional on Y_n and X_n, where $Y_n = \{y_1, \ldots, y_n\}$ and $X_n = \{x_1, \ldots, x_n\}$. Thus, the smoothed state vector $\widehat{a}_t = E(\alpha_t \mid Y_n, X_n)$ and the smoothed state variance matrix $\widehat{P}_t = Var(\alpha_t \mid Y_n, X_n)$ are estimated using all the observations for $t = 1, \ldots, n$. In order to account for the diffuse initialization of α_1, we use the exact initial state smoothing algorithm suggested by Koopman and Durbin (2003).

A.3 Bayesian Analysis

In principle, the integral in equation (22) can be evaluated numerically by drawing a sample of n random draws of ψ, denoted $\psi^{(i)}$ with $i = 1, \ldots, n$, from $p(\psi \mid y, x)$ and then estimating $\overline{g}$ by the sample mean of $g(\psi)$. As $p(\psi \mid y, x)$ is not a density with known analytical properties, such a direct sampling method is not feasible, though. Therefore, we switch to importance sampling. The idea is to use an importance density $g(\psi \mid y, x)$ as a proxy for $p(\psi \mid y, x)$, where $g(\psi \mid y, x)$ should be chosen as a distribution that can be simulated directly and is as close to $p(\psi \mid y, x)$ as possible. By Bayes' theorem and after some manipulations, equation (22) can be rewritten as

$$\overline{g} = \frac{\int g(\psi)\, z^g(\psi, y, x)\, g(\psi \mid y, x)\, d\psi}{\int z^g(\psi, y, x)\, g(\psi \mid y, x)\, d\psi} \tag{A.5}$$

with

$$z^g(\psi, y, x) = \frac{p(\psi)\, p(y \mid \psi)}{g(\psi \mid y, x)} \tag{A.6}$$

Using a sample of n random draws $\psi^{(i)}$ from $g(\psi \mid y, x)$, an estimate $\overline{g}_n$ of $\overline{g}$ can then be obtained as

$$\overline{g}_n = \frac{\sum_{i=1}^{n} g\left(\psi^{(i)}\right) z^g\left(\psi^{(i)}, y, x\right)}{\sum_{i=1}^{n} z^g\left(\psi^{(i)}, y, x\right)} = \sum_{i=1}^{n} w_i g\left(\psi^{(i)}\right) \tag{A.7}$$

with w_i

$$w_i = z^g\left(\psi^{(i)}, y, x\right) / \sum_{i=1}^{n} z^g\left(\psi^{(i)}, y, x\right) \tag{A.8}$$

the weighting function reflecting the importance of the sampled value $\psi^{(i)}$ relative to other sampled values.

Geweke (1989) shows that if $g(\psi \mid y, x)$ is proportional to $p(\psi \mid y, x)$, and under a number of weak regularity conditions, $\overline{g}_n$ will be a consistent estimate of $\overline{g}$ for $n \to \infty$.

A.4 Computational Aspects of Importance Sampling

As a first step importance density $g(\psi \mid y, x)$, we take a large sample normal approximation to $p(\psi \mid y, x)$, i.e.

$$g(\psi \mid y, x) = N\left(\widehat{\psi}, \widehat{\Omega}\right) \tag{A.9}$$

where $\widehat{\psi}$ is the mode of $p(\psi \mid y, x)$ obtained from maximizing

$$\log p(\psi \mid y, x) = \log p(y \mid \psi) + \log p(\psi) - \log p(y) \tag{A.10}$$

with respect to $\widehat{\psi}$ and where $\widehat{\Omega}$ denotes the covariance matrix of $\widehat{\psi}$. Note that $p(y \mid \psi)$ is given by the likelihood function derived from the exact Kalman filter and we do not need to calculate $p(y)$ as it does not depend on ψ.

In drawing from $g(\psi \mid y, x)$, efficiency was improved by the use of antithetic variables, i.e. for each $\psi^{(i)}$ we take another value $\widetilde{\psi}^{(i)} = 2\widehat{\psi} - \psi^{(i)}$, which is equiprobable with $\psi^{(i)}$. This results in a simulation sample that is balanced for location (Durbin and Koopman 2001).

As any numerical integration method delivers only an approximation to the integrals in equation $(A.5)$, we monitor the quality of the approximation by estimating the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ (Bauwens, Lubrano and Richard 1999, chap. 3, eq. 3.34). This error bound represents a 95% confidence interval for the percentage deviation of $\overline{g}_n$ from $\overline{g}$. It should not exceed 10%. In practice this can be achieved by increasing n, except when the coefficient of variation of the weights w_i is unstable as n increases. An unstable coefficient of variation of w_i signals poor quality of the importance density. This was exactly the problem encountered in the empirical analysis.

Note that the normal approximation in equation $(A.9)$ selects $g(\psi \mid y, x)$ in order to match the location and covariance structure of $p(\psi \mid y, x)$ as good as possible. One problem is that the normality assumption might imply that $g(\psi \mid y, x)$ does not match the tail behavior of $p(\psi \mid y, x)$. If $p(\psi \mid y, x)$ has thicker tails than $g(\psi \mid y, x)$, a draw $\psi^{(i)}$ from the tails of $g(\psi \mid y, x)$ can imply an explosion of $z^g\left(\psi^{(i)}, y, x\right)$. This is due to a very small value for $g(\psi \mid y, x)$ being associated with a relatively large value for $p(\psi)\, p(y \mid \psi)$, as the latter is proportional to $p(\psi \mid y, x)$. Importance sampling is inaccurate in this case as this would lead to a weight w_i close to one, i.e. $\overline{g}_n$ is determined by a single draw $\psi^{(i)}$. This is signaled by instability of the weights and a probabilistic error bound that does not decrease in n.

In order to help prevent explosion of the weights, we change the construction of the importance density in two respects (Bauwens *et al.* 1999, chap. 3). First, we inflate the approximate covariance matrix $\widehat{\Omega}$ a little. This reduces the probability that $p(\psi \mid y, x)$ has thicker tails than $g(\psi \mid y, x)$. Second, we use a sequential updating algorithm for the importance density. This algorithm starts from the importance density defined by $(A.9)$, with inflation of $\widehat{\Omega}$, estimates posterior moments for $p(\psi \mid y, x)$ and then defines a new importance density from these estimated moments. This improves the estimates for $\widehat{\psi}$ and $\widehat{\Omega}$. We continue updating the importance density until the weights stabilize. The number of importance samples n was chosen to make sure that the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ does not exceed 10%.

Appendix B: Data

The sample of all data we use runs from 1970:2 to 2005:4.

- **Inflation**: quarterly inflation rate, defined as $400(lnP_t - lnP_{t-1})$, with P_t the seasonally adjusted quarterly GDP deflator. Sources: Area Wide Model (Fagan et al, 2005) and Bank for International Settlements;

- **Real output**: quarterly output, defined as $100ln(GDP_t)$, with GDP_t the seasonally adjusted quarterly GDP in constant prices. Sources: Area Wide Model (Fagan et al, 2005) and Bank for International Settlements;

- **Key interest rate**: quarterly central bank key interest rate. Sources: European Central Bank and Bank for International Settlements.